Exploring the oscillatory Unruh effect for N accelerating detectors inside a cavity

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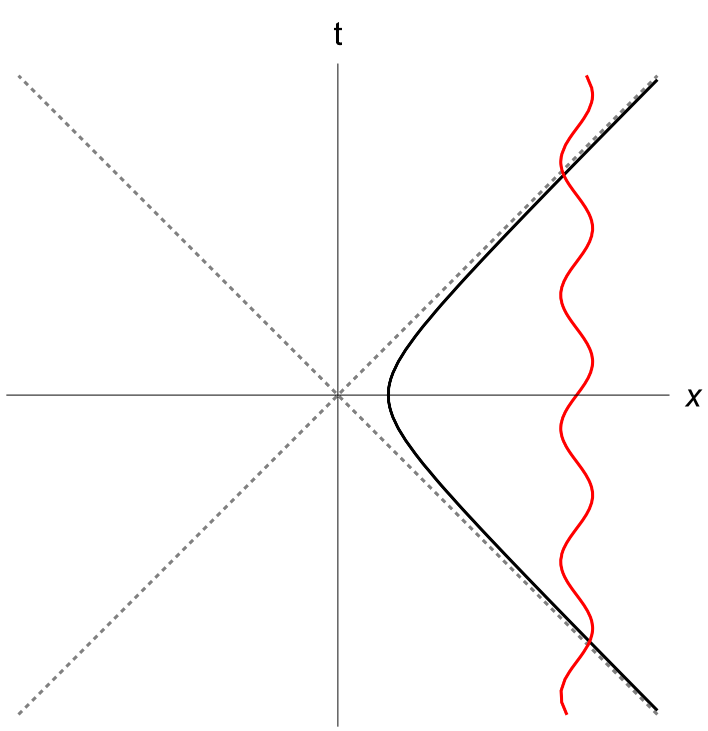

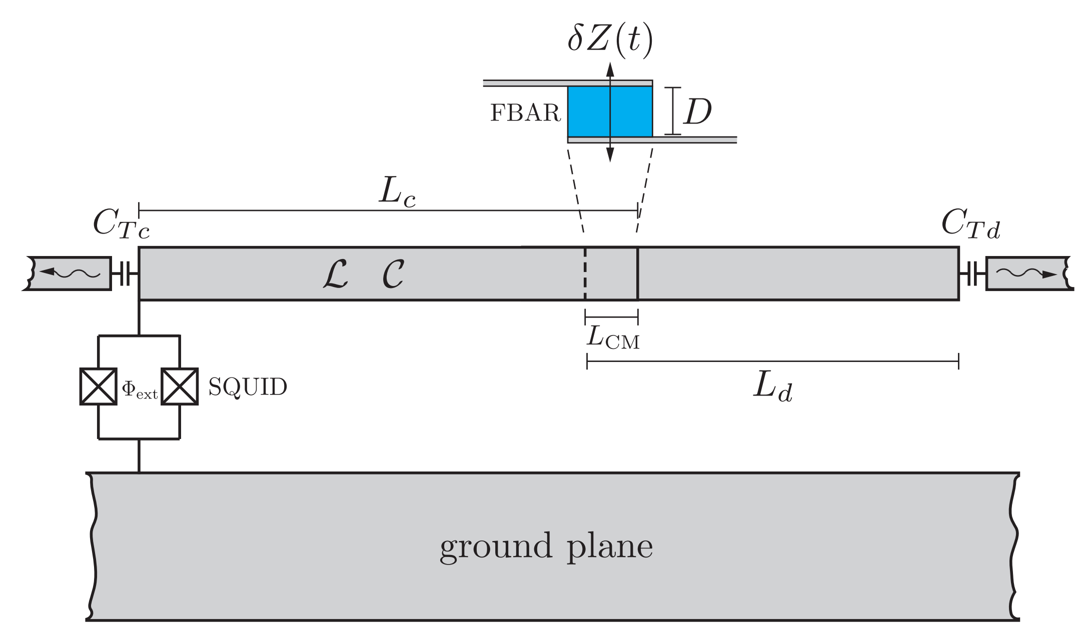

Unruh effect

Minkowski Vacuum $\xrightarrow{\text{accelerating observer}}$ Thermal State: $T = \frac{\hbar a}{2\pi c k_B}$

T=1K: a proper acceleration of $2.47 \times 10^{20}\ \mathrm{m/s^2}$ would be required

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Enhancing Acceleration Radiation from Ground-State Atoms via Cavity Quantum Electrodynamics

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When ground-state atoms are accelerated through a high Q microwave cavity, radiation is produced with an intensity which can exceed the intensity of Unruh acceleration radiation in free space by many orders of magnitude. The reason is a strong nonadiabatic effect at cavity boundaries and its interplay with the standard Unruh effect. The cavity field at steady state is still described by a thermal density matrix under most conditions. However, under some conditions gain is possible, and when the atoms are injected in a regular fashion, squeezed radiation can be produced.

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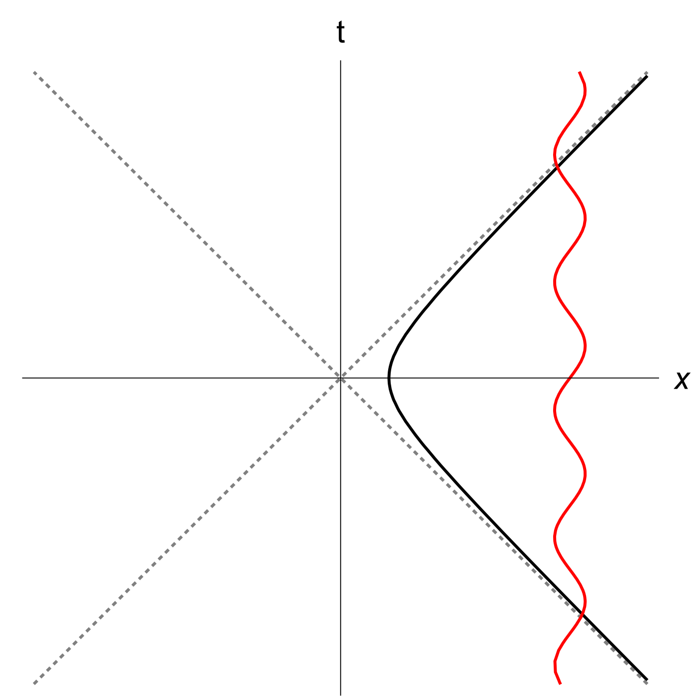
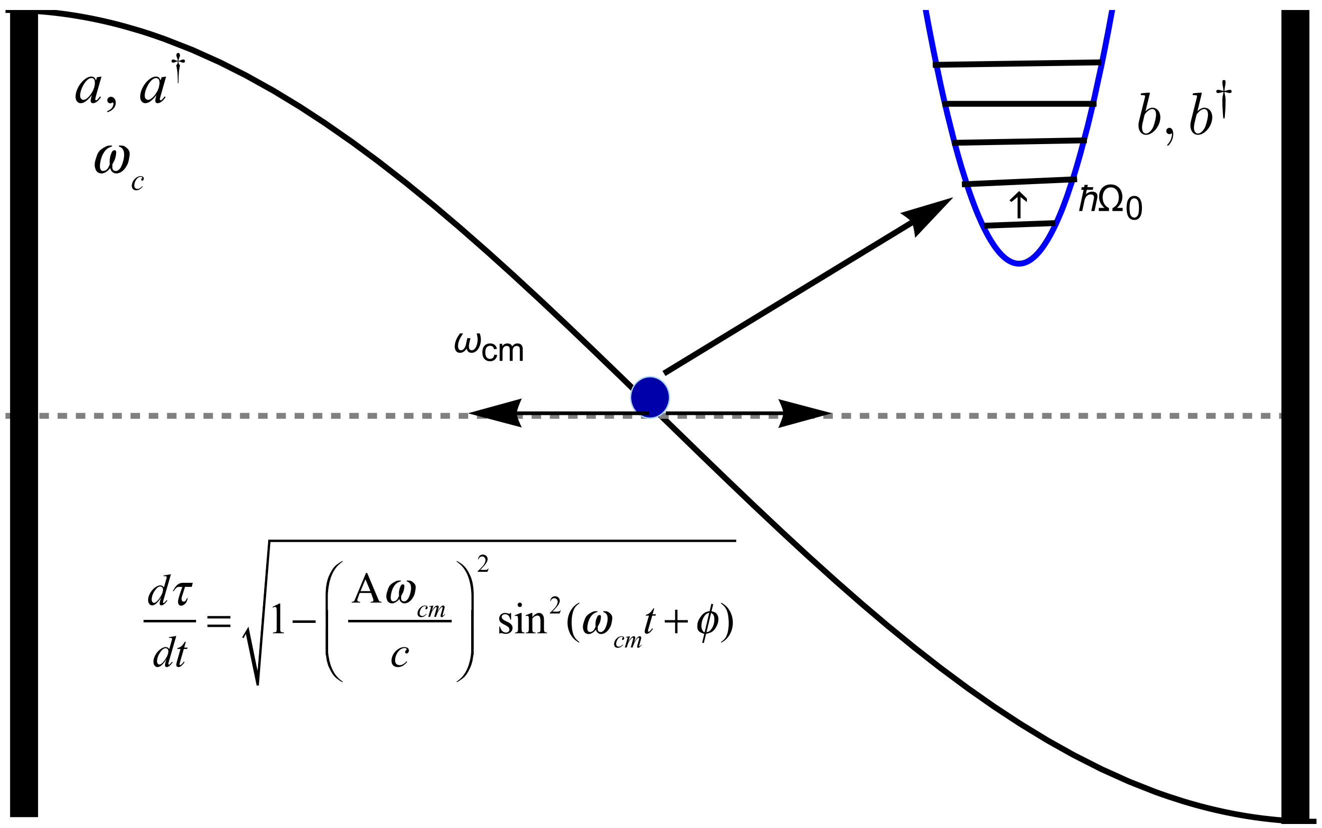


$$x(t) = A\cos(\omega_{cm} t + \phi)$$

$$\ddot{x}_{max} = A\omega_{cm}^2$$

The cavity-detector system



Apply single mode approximation to the cavity photon field

A point like detector is linearly coupled to the cavity

The detector's internal degree of freedom is modeled as a harmonic oscillator of bare natural frequency Ω_0

$$S=-\int d^{1+1}x\,\frac{1}{2}\partial_\mu\Phi\partial^\mu\Phi+\int d\tau\left\{\frac{m_0}{2}\left[(\partial_\tau Q)^2-\Omega_0^2Q^2\right]+\lambda_0\int d^{1+1}x\,Q(\tau)\Phi(t,x)\delta^{1+1}(x^\mu-x^\mu(\tau))\right\}$$

$$H(t)/\hbar\omega_c=a^\dagger a+\Omega_0\frac{d\tau}{dt}b^\dagger b-\lambda_0\frac{d\tau}{dt}\sin\left[k_cA\cos(\omega_{CM}t+\phi)\right](a^\dagger+a)(b^\dagger+b)$$

Hamiltonian in the lab frame!

Can accurately map relativistic Hamiltonian to simple NDPA model:

$$H(t)/\hbar\omega_c = a^\dagger a + \Omega_0 \frac{d\tau}{dt} b^\dagger b + \lambda_0 \frac{d\tau}{dt} \sin\left[k_c A \cos(\omega_{CM} t + \phi)\right](a^\dagger + a)(b^\dagger + b)$$

$$\approx a^\dagger a + \Omega_0 \left[D_0 + D_2 \cos(2\omega_{CM} t)\right] b^\dagger b - \lambda_0 C_1 \cos(\omega_{CM} t)(a^\dagger + a)(b^\dagger + b)$$

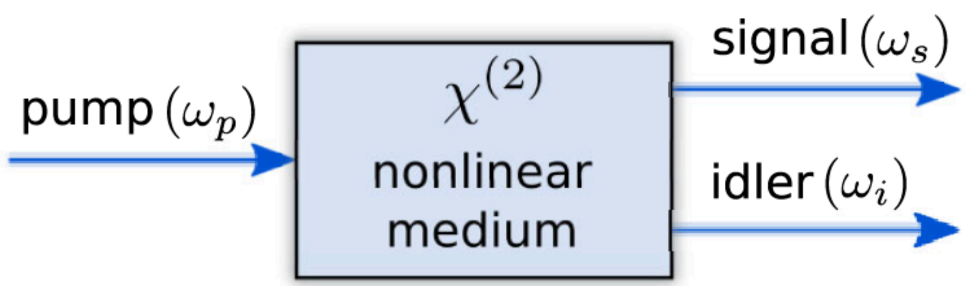
Renormalized detector frequency

$$\omega_c + \Omega_0 D_0 = \omega_{CM} \xrightarrow[\text{Interaction Picture}]{\text{Rotating Wave Approximation}} H_I = -\lambda(a^\dagger b^\dagger + ab)$$

Non-Degenerate Parametric Amplification (NDPA)

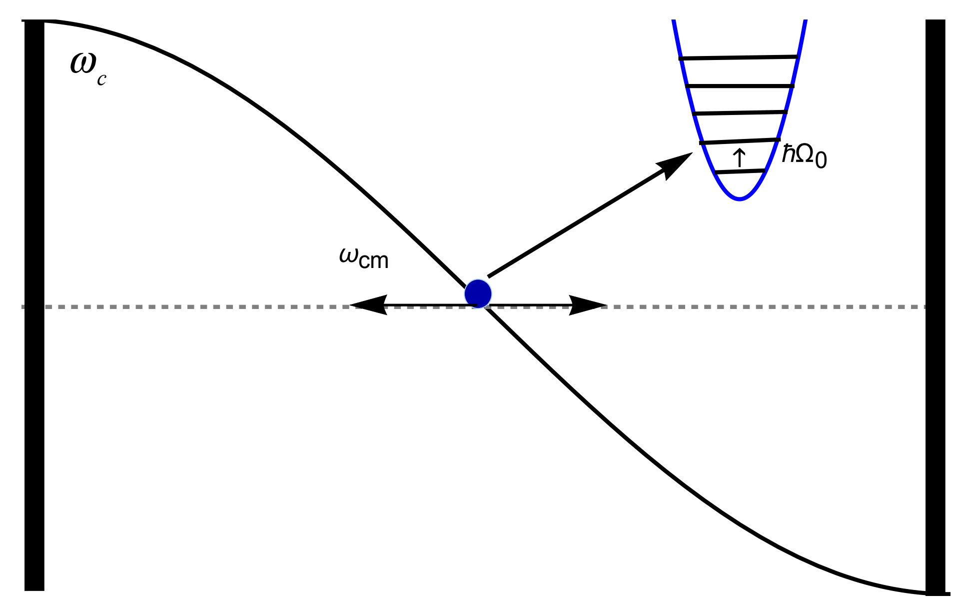
VS

Oscillatory Unruh Effect



A pump photon is down-converted by a nonlinear medium into a signal and an idler photon, whose frequencies add up to that of the pump photon.

$$\omega_s + \omega_i = \omega_p$$



center-of-mass oscillation: pump
cavity/detector modes: signal/idler modes

$$\omega_c + \Omega_0 D_0 = \omega_{CM}$$

Quantum Langevin Equation: Include the interaction between cavity/detector modes and the environment

$a \rightarrow$ cavity, $b \rightarrow$ detector $\qquad \dfrac{d\hat{a}}{dt} = \dfrac{i}{\hbar}\left[\hat{H}, \hat{a}\right] - \dfrac{1}{2}\gamma\hat{a} - i\sqrt{\gamma}\,\hat{a}_{\text{in}}, \quad a \leftrightarrow b.$

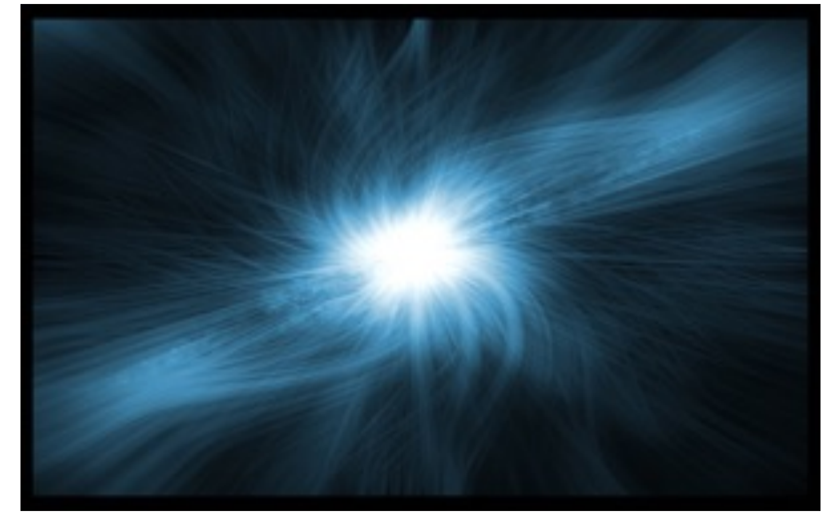
Environment temperature T=0: $\langle \hat{a}_{\text{in}}^{\dagger}(t)\hat{a}_{\text{in}}(t')\rangle = 0, \quad \langle \hat{a}_{\text{in}}(t)\hat{a}_{\text{in}}^{\dagger}(t')\rangle = \delta(t-t')$

$$\langle \hat{b}_{\text{in}}^{\dagger}(t)\hat{b}_{\text{in}}(t')\rangle = 0, \quad \langle \hat{b}_{\text{in}}(t)\hat{b}_{\text{in}}^{\dagger}(t')\rangle = \delta(t-t')$$

Gaussian state: Described by first and second order moments

$$\langle \hat{a} \rangle = \langle \hat{b} \rangle = 0$$

Let there be light!



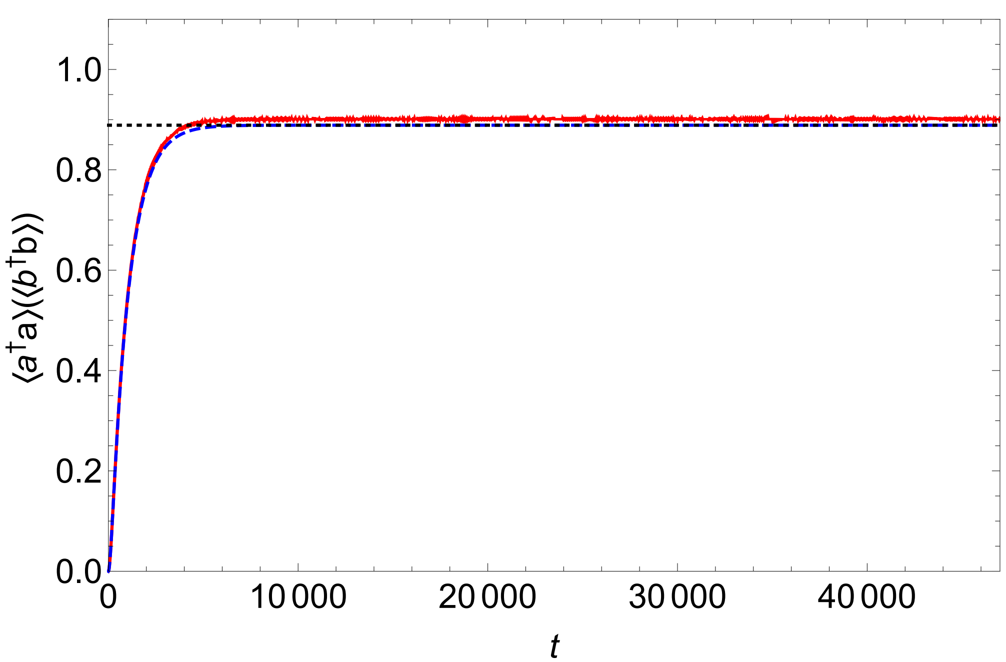
$$\langle a^\dagger(t)a(t)\rangle = \langle b^\dagger(t)b(t)\rangle \xrightarrow[\quad]{t\to\infty} \frac{2\lambda^2}{\gamma^2-4\lambda^2} = \frac{2\eta^2}{1-4\eta^2}$$

Parametric Stability: $\eta = \frac{\lambda}{\gamma} < \eta_{\text{crit}} = \frac{1}{2}$

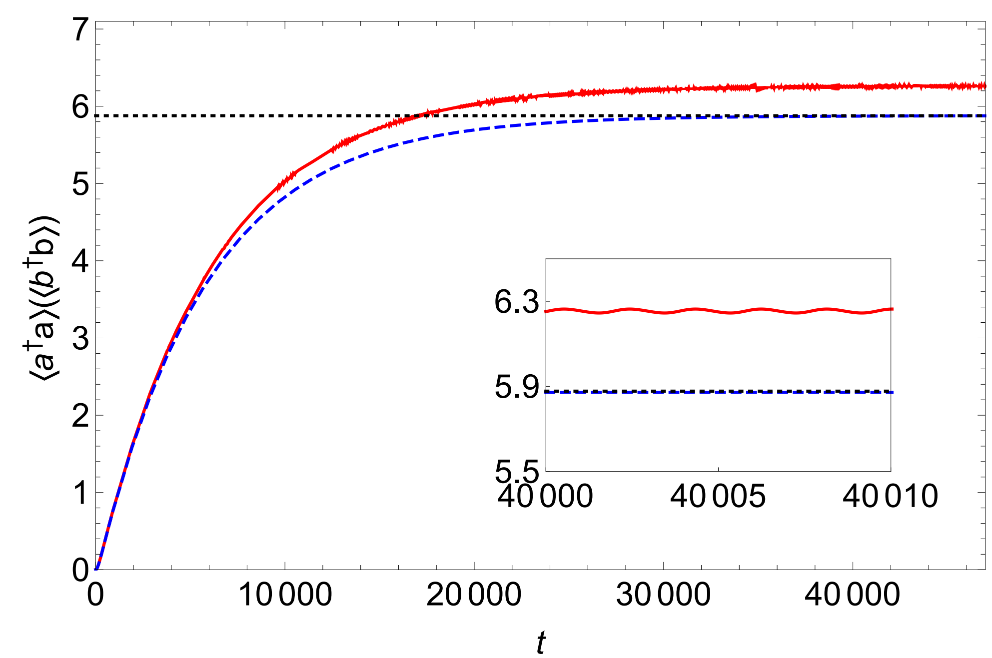
Full Hamiltonian solution

NDPA solution

Steady state solution



$\eta = 0.40$



$\eta = 0.48$

Example relativistic parameter values: $\frac{v_{\max}}{c} = 0.8, \lambda_0 = 0.01$

Two Mode Gaussian State

$\langle a \rangle = \langle a^\dagger \rangle = 0 \qquad \langle a^2 \rangle = \langle (a^\dagger)^2 \rangle = 0$

↓

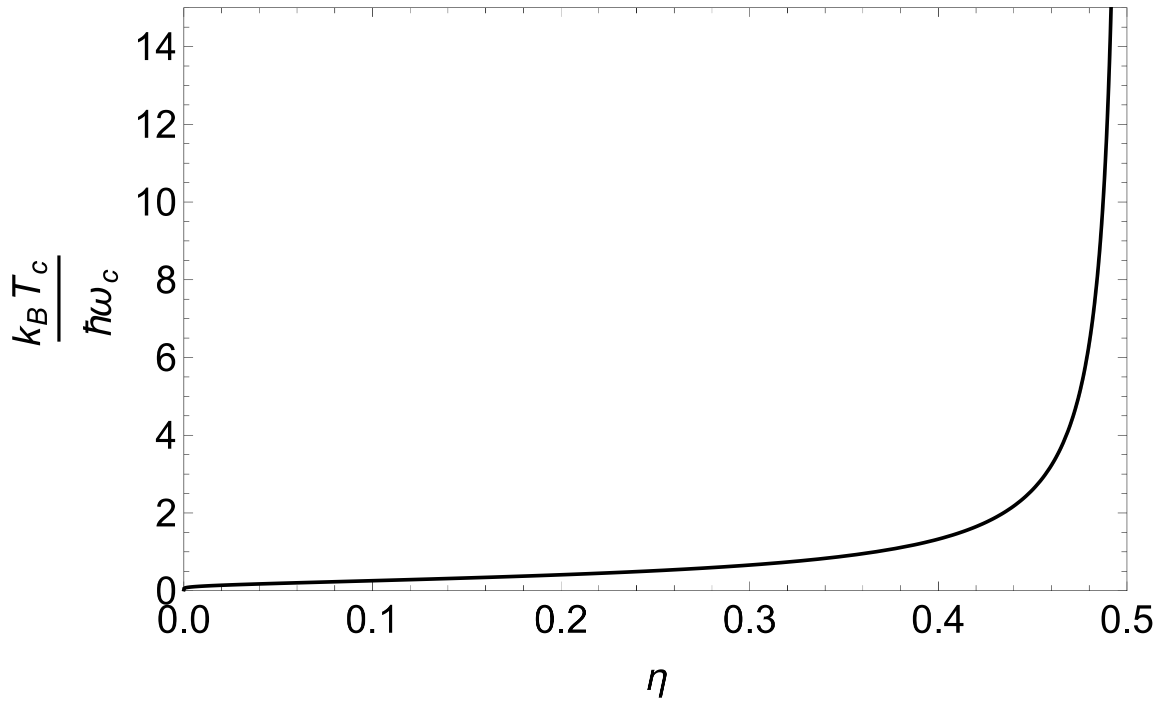
zero-mean, circularly-symmetric
in phase space

↓

Effective Thermal State:

$$\frac{k_B T_c(t)}{\hbar\omega_c} \underset{t \to \infty}{=\!=} \left[\ln\left(\frac{1-2\eta^2}{2\eta^2} \right) \right]^{-1}$$

$\eta = \frac{\lambda}{\gamma} < \frac{1}{2}$



Near instability: $\frac{k_B T_c}{\hbar\omega_c} \gg 1$

Circuit QED analogue

film bulk acoustic resonator driven with $\delta Z(t) = \delta Z_0 \cos(\omega_{\mathrm{CM}} t)$

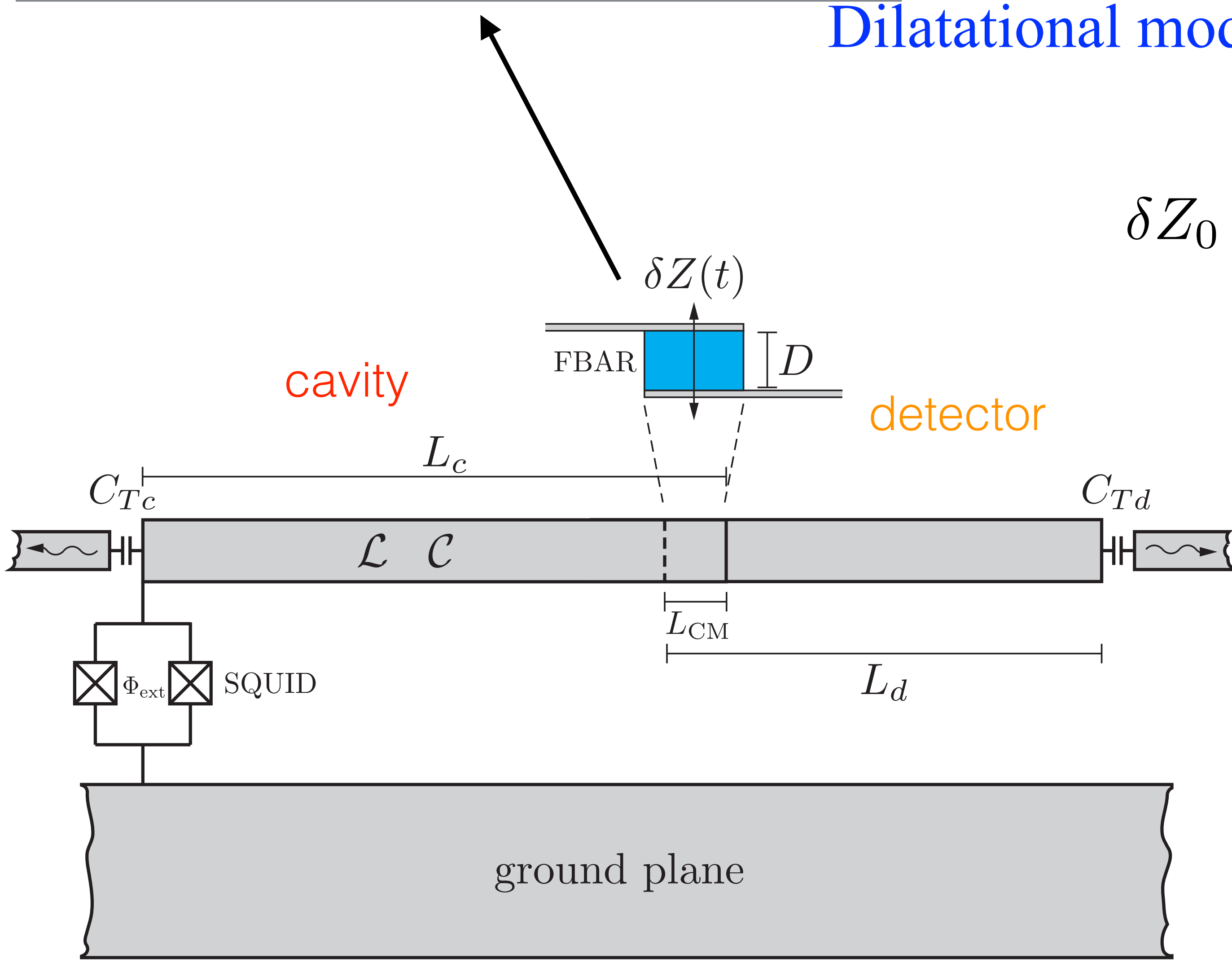
FBAR thickness $D \sim 500 nm$

longitudinal speed of sound in silicon $v_l \sim 10^4 \;\mathrm{m/s}$

Dilatational mode frequency of FBAR $\omega_{\mathrm{CM}} = \frac{\pi v_l}{D} \sim 2\pi \times 10 \;\mathrm{GHz}$

$$\delta Z_0 = 10^{-11}\mathrm{m},\; a_{\max} \sim 10^{10}\mathrm{m/s^2},\; v_{\max} \sim 1\mathrm{m/s}$$

non-relativistic, but photon production from vacuum still possible!



Actual system: detector inside cavity
Analogue system: detector outside cavity

$$v = \frac{1}{\sqrt{\mathcal{LC}}} \sim 10^8 \;\mathrm{m/s}$$

$$\omega_c = \frac{\pi}{L_c\sqrt{\mathcal{LC}}}, \quad \omega_d = \frac{\pi}{L_d\sqrt{\mathcal{LC}}}$$

[C. M. Wilson *et al.*, Nature **479**, 377 (2011)]
[M. Sanz *et al.,* Quantum **2**, 91 (2018)]

Circuit QED analogue

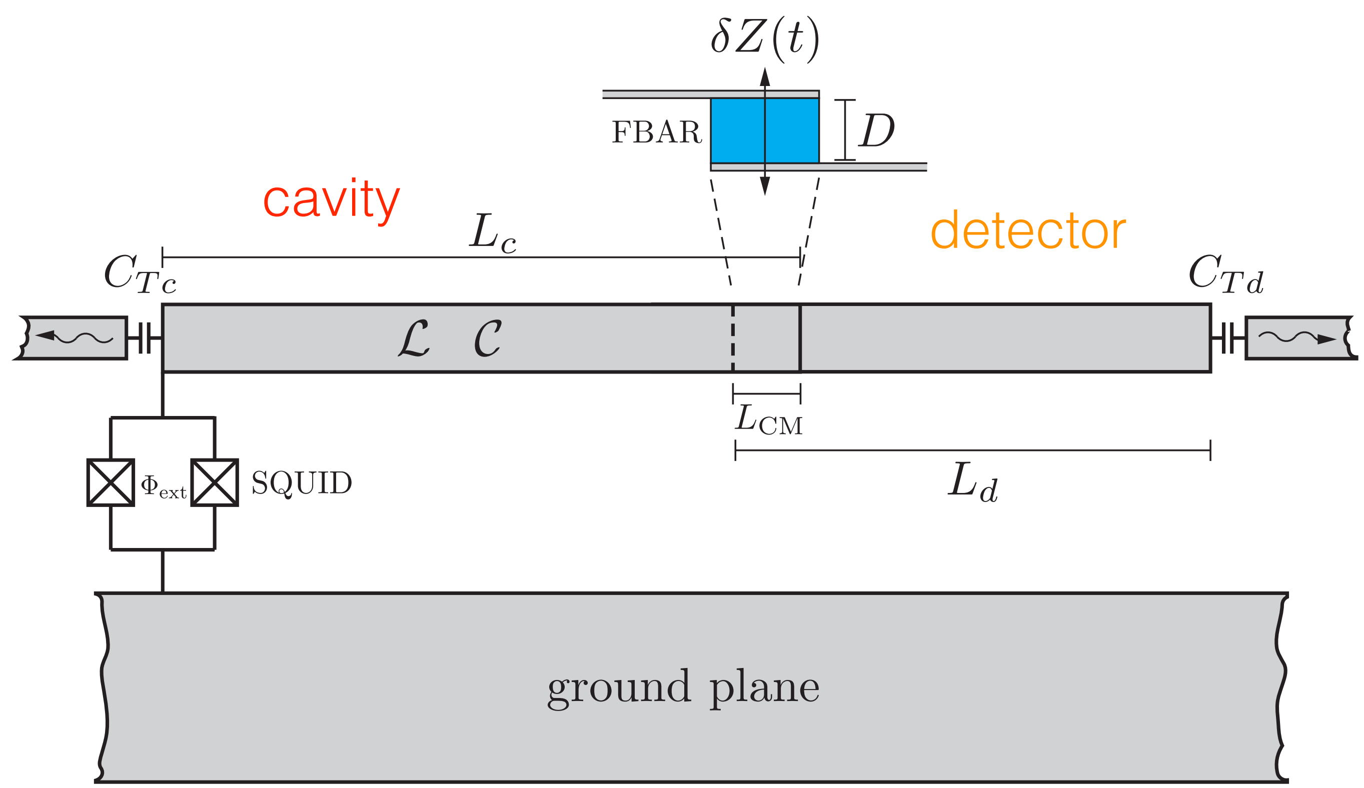
$$H = \sum_n \hbar\omega_n \hat{a}_n^\dagger \hat{a}_n - \frac{\delta Z(t)}{D} \sum_{n,n'} \lambda_{nn'} \left(\hat{a}_n - \hat{a}_n^\dagger\right)\left(\hat{a}_{n'} - \hat{a}_{n'}^\dagger\right)$$

$$\lambda_{nn'} = \hbar\sqrt{\omega_n \omega_{n'}} \left(\frac{\pi}{\Phi_0}\right)^2 \frac{\mathcal{C}_{\mathrm{CM}}^{(0)}}{\sqrt{C_n C_{n'}}} \int_0^{L_{\mathrm{CM}}} dx \left[\Phi_{d,n}(L_d - x) - \Phi_{c,n}(L_c - x)\right]\left[\Phi_{d,n'}(L_d - x) - \Phi_{c,n'}(L_c - x)\right]$$

ω_1, ω_2 : fundamental normal mode frequencies of the cavity-detector system

$$\omega_{\mathrm{CM}} = \omega_1 + \omega_2$$

$$\hat{H}_I = -\frac{\delta Z(0)}{D} \lambda_{12} \left(\hat{a}_1 \hat{a}_2 + \hat{a}_1^\dagger \hat{a}_2^\dagger\right)$$

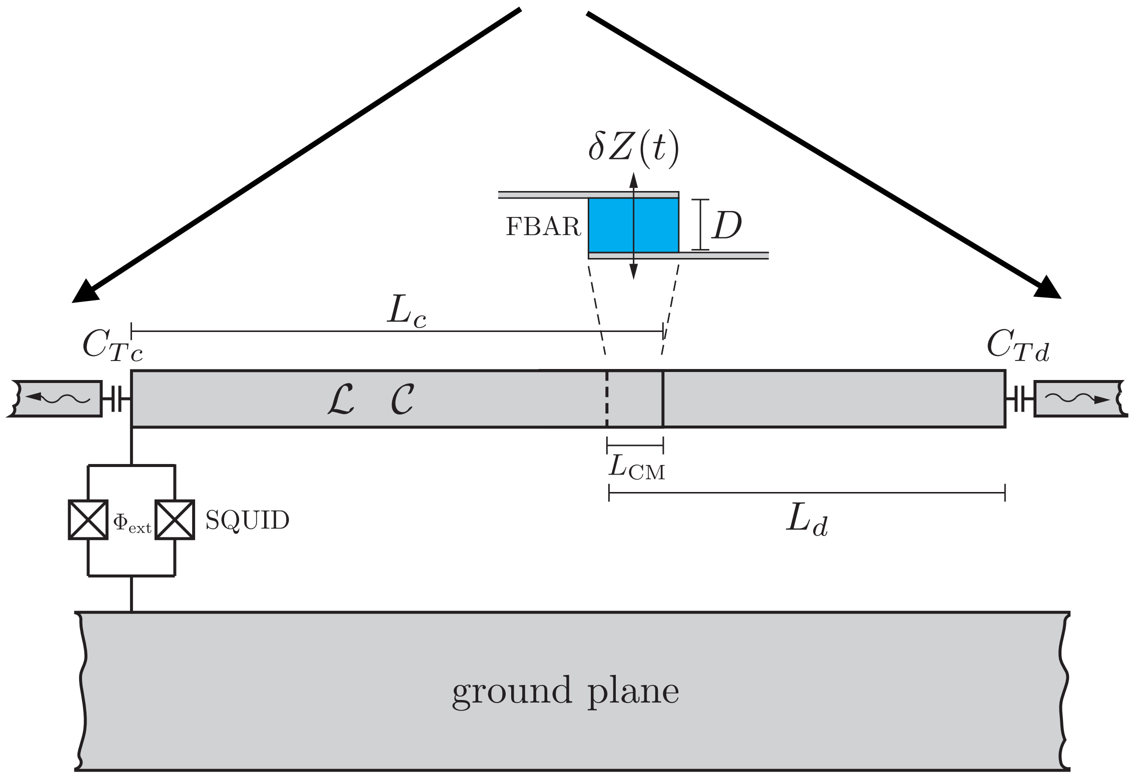


$$\lambda_{12}^{\max} = 0.051\hbar\omega_1 \text{ for } \omega_1 = 2\pi \times 3.84 \text{ GHz}, \omega_2 = 2\pi \times 5.7 \text{ GHz}$$

$$L_{\mathrm{CM}} = 87\mu\mathrm{m}, \quad L_{\mathrm{c}} = 1.1\mathrm{cm}, \quad L_{\mathrm{d}} = 0.77\mathrm{cm}$$

Measurement

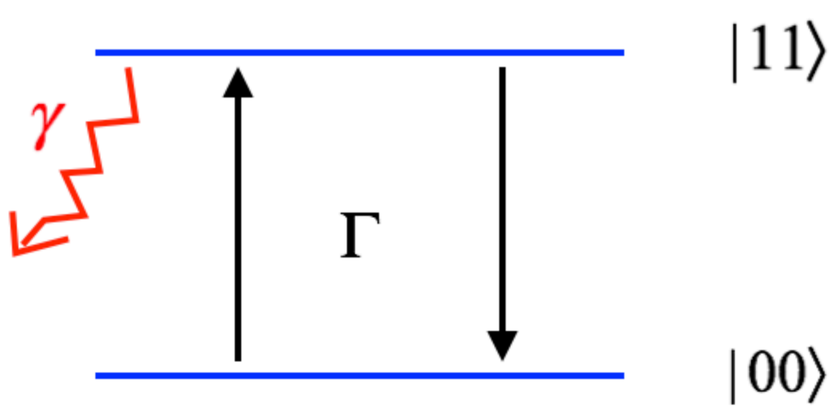
Transmission line induced dissipation γ_c, γ_d



$$\hat{H}_I = -\frac{\delta Z(0)}{D}\lambda_{12}\left(\hat{a}_1\hat{a}_2 + \hat{a}_1^\dagger\hat{a}_2^\dagger\right), \text{ define } \lambda = \frac{\delta Z(0)}{D}\lambda_{12}$$

$$\lambda \propto L_{CM} \text{ for } L_{CM} \ll L_c$$

In long-time limit: detailed balance



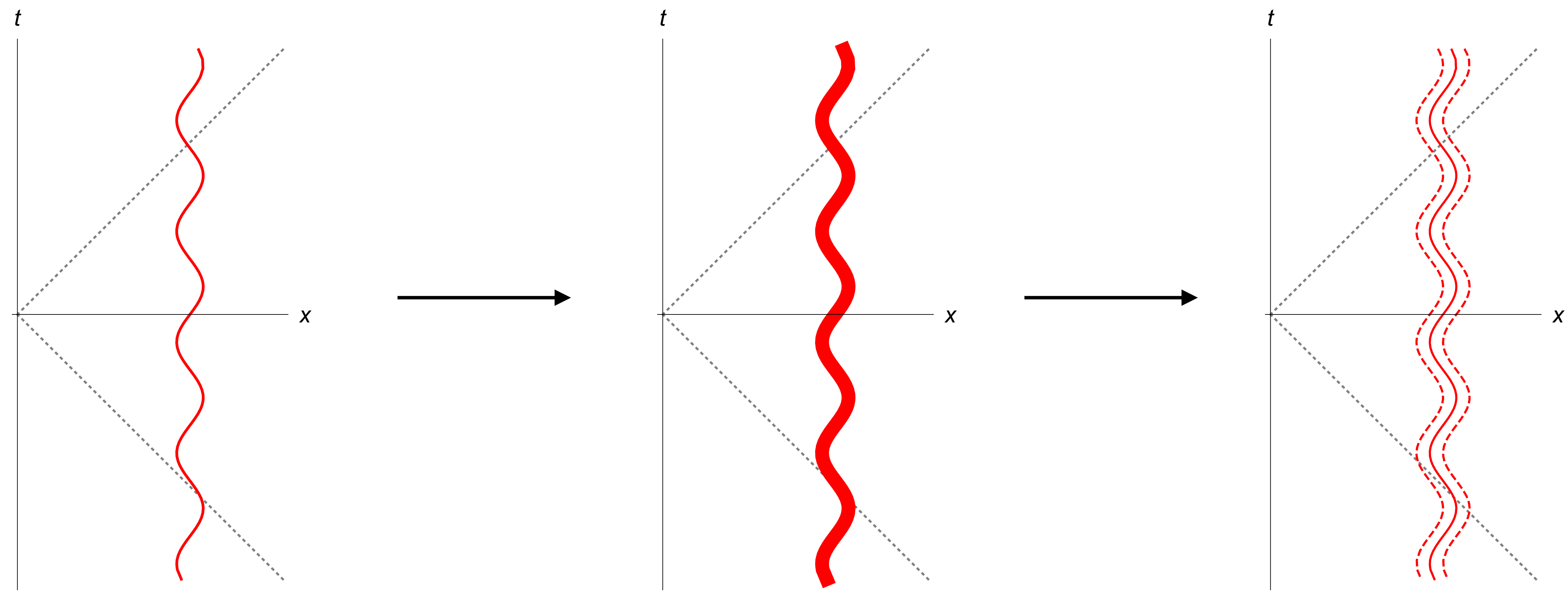
photon pair production rate:

$$\Gamma = \left(\frac{\lambda}{\hbar}\right)^2 \frac{4}{\gamma_c + \gamma_d} \propto L_{CM}^2, \quad \boxed{\Gamma_{max} \sim 10^3/s}$$

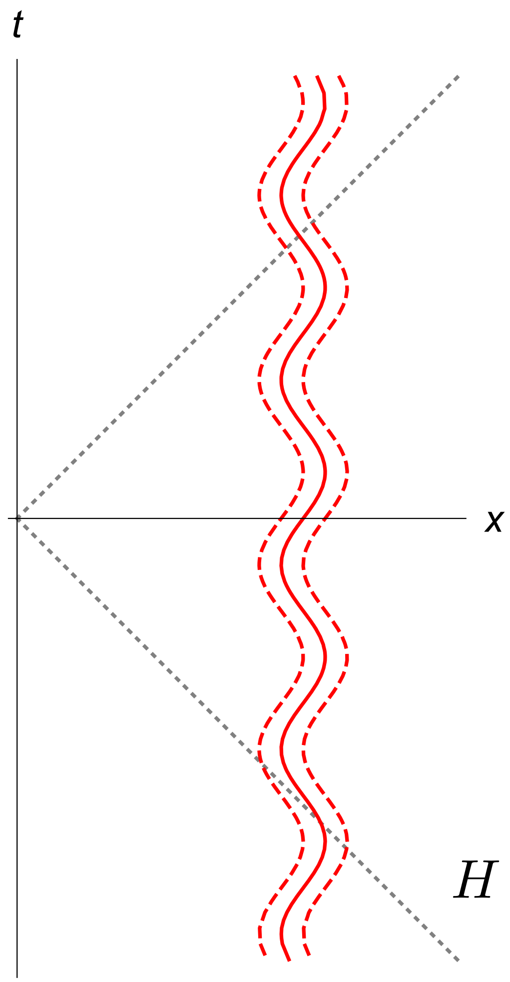
Assuming realizable quality factor $Q \sim 10^5$
and maximum coupling strength $\lambda/\hbar \sim 10^4$ Hz

Present work: increasing λ
by increasing L_{CM}, $\lambda \propto L_{\mathrm{CM}}$

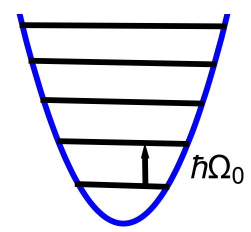
Future work: consider $N > 1$
independent detectors, $\lambda \propto \sqrt{N}$

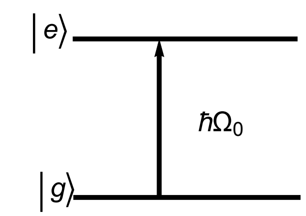


Future Work



One Harmonic Oscillator detector → N>>1 TLS detectors





$$H = \omega_c a^\dagger a + \Omega_0 \frac{d\tau}{dt} \sum_{i=1}^{N} \sigma_{iz} + \lambda_0 \frac{d\tau}{dt} \cos k_c \left[l + A \cos\left(\omega_{cm} t + \phi\right)\right] \left(a^\dagger + a\right) \sum_{i=1}^{N} \left(\sigma_{i+} + \sigma_{i-}\right)$$

Parametric Instability
(oscillator detector) → Phase Transition
(N>>1 TLS detectors)

Photon emission (production) rate:

Normal phase
$\Gamma \propto N$

→

Superradiant phase
$\Gamma \propto N^2$?

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[H. Wang *et al.*, Phys. Rev. A **99**, 053833 (2019)]

Linear differential equation for second order moments

$$\frac{d\hat{a}}{dt} = \frac{i}{\hbar}\left[\hat{H}, \hat{a}\right] - \frac{1}{2}\gamma\hat{a} - i\sqrt{\gamma}\,\hat{a}_{\text{in}}, \quad \langle \hat{a}^{\dagger}_{\text{in}}(t)\hat{a}_{\text{in}}(t')\rangle = 0, \quad \langle \hat{a}_{\text{in}}(t)\hat{a}^{\dagger}_{\text{in}}(t')\rangle = \delta(t-t'), \quad a \leftrightarrow b.$$

$$\frac{d}{dt}\begin{bmatrix} \langle aa\rangle \\ \langle a^\dagger a\rangle \\ \langle a^\dagger a^\dagger\rangle \\ \langle ab\rangle \\ \langle a^\dagger b\rangle \\ \langle ab^\dagger\rangle \\ \langle a^\dagger b^\dagger\rangle \\ \langle bb\rangle \\ \langle b^\dagger b\rangle \\ \langle b^\dagger b^\dagger\rangle \end{bmatrix} = \begin{bmatrix} -\gamma & 0 & 0 & 0 & 0 & 2i\lambda & 0 & 0 & 0 & 0 \\ 0 & -\gamma & 0 & -i\lambda & 0 & 0 & i\lambda & 0 & 0 & 0 \\ 0 & 0 & -\gamma & 0 & -2i\lambda & 0 & 0 & 0 & 0 & 0 \\ 0 & i\lambda & 0 & -\gamma & 0 & 0 & 0 & 0 & i\lambda & 0 \\ 0 & 0 & i\lambda & 0 & -\gamma & 0 & 0 & -i\lambda & 0 & 0 \\ -i\lambda & 0 & 0 & 0 & 0 & -\gamma & 0 & 0 & 0 & i\lambda \\ 0 & -i\lambda & 0 & 0 & 0 & 0 & -\gamma & 0 & -i\lambda & 0 \\ 0 & 0 & 0 & 0 & 2i\lambda & 0 & 0 & -\gamma & 0 & 0 \\ 0 & 0 & 0 & -i\lambda & 0 & 0 & i\lambda & 0 & -\gamma & 0 \\ 0 & 0 & 0 & 0 & 0 & -2i\lambda & 0 & 0 & 0 & -\gamma \end{bmatrix} \begin{bmatrix} \langle aa\rangle \\ \langle a^\dagger a\rangle \\ \langle a^\dagger a^\dagger\rangle \\ \langle ab\rangle \\ \langle a^\dagger b\rangle \\ \langle ab^\dagger\rangle \\ \langle a^\dagger b^\dagger\rangle \\ \langle bb\rangle \\ \langle b^\dagger b\rangle \\ \langle b^\dagger b^\dagger\rangle \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ i\lambda \\ 0 \\ 0 \\ -i\lambda \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\frac{d\vec{V}(t)}{dt} = M(t)\vec{V}(t) + \vec{K}$$

$$\vec{V}(t) = \int_0^t dt' e^{M(t-t')}\vec{K} + e^{Mt}\vec{V}(0)$$

non-zero second order moments

$$\langle a^\dagger(t)a(t)\rangle = \langle b^\dagger(t)b(t)\rangle = -\frac{\lambda e^{-t(\gamma+2\lambda)}\left[\gamma\left(e^{4\lambda t}-1\right)+2\lambda\left(-2e^{t(\gamma+2\lambda)}+e^{4\lambda t}+1\right)\right]}{2(\gamma^2-4\lambda^2)}$$

$$\langle a^\dagger(t)b^\dagger(t)\rangle = -\langle a(t)b(t)\rangle = \frac{i\lambda e^{-t(\gamma+2\lambda)}\left[\gamma\left(-2e^{t(\gamma+2\lambda)}+e^{4\lambda t}+1\right)+2\lambda\left(e^{4\lambda t}-1\right)\right]}{2(\gamma^2-4\lambda^2)}$$

Verification of quantum pair production

E.g. Two-mode squeezing

[J. R. Johansson *et al.*, Phys. Rev. A **87**, 043804 (2013)]

superposition quadrature operators:

$$X_1(t) = 2^{-3/2}\left[e^{i\omega_1 t}a_1(t) + e^{-i\omega_1 t}a_1^\dagger(t) + e^{i(\omega_2 t-\theta)}a_2(t) + e^{-i(\omega_2 t-\theta)}a_2^\dagger(t)\right]$$

$$X_2(t) = -2^{-3/2}i\left[e^{i\omega_1 t}a_1(t) - e^{-i\omega_1 t}a_1^\dagger(t) + e^{i(\omega_2 t-\theta)}a_2(t) - e^{-i(\omega_2 t-\theta)}a_2^\dagger(t)\right]$$

Coherent States: $\Delta X_1 = \Delta X_2 = 1/2$

Quantum squeezed states: $\min_\theta \Delta X_1 < 1/2$

For considered realistic device parameters, quantum squeezing for $T < 70\mathrm{mK}$