MODIFIED UNRUH EFFECT FROM GENERALIZED UNCERTAINTY PRINCIPLE

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Research on generalizations of the Heisenberg uncertainty principle started soon after Heisenberg 1927 paper...

- M. Bronstein, 1935 - C.N. Yang, 1947 - H.Snyder, 1947 - F. Karolyhazy, 1966.
- C.A. Mead, 1964: "***To reduce uncertainty in the position of a particle, high energy photons are needed. But high energy photons carry strong gravitational fields, which tend to move the particle***".
- **String Theory** (Veneziano 1987, Gross 1987) investigated Gedanken experiments on high energy string scattering involving Gravity.**To larger momentum transfer not always correspond shorter distances**. The "effective" stringy uncertainty relation should read

$$q \sim \frac{\hbar}{p} + Y\lambda_s^2 \frac{p}{\hbar}$$

p = momentum transfer

q = impact parameter

F.Scardigli (1999): **Measure process in presence of gravity**.

Probing a region of size **Δx** produce quantum fluctuations in the metric field with an amplitude in energy of

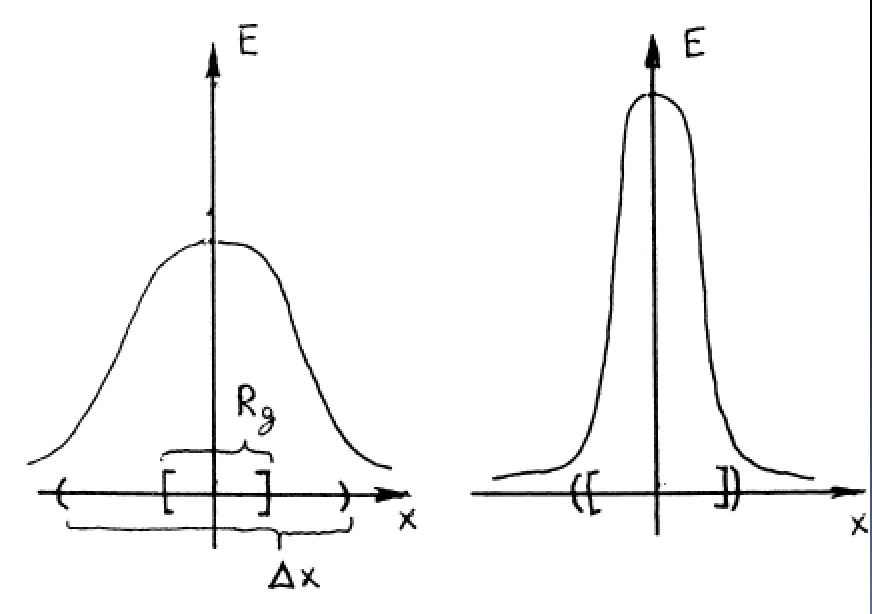
$$\Delta E \sim \frac{\hbar c}{2\Delta x}$$

Usually $R_g(\Delta E) \ll \Delta x$, where $R_g(\Delta E) = 2G\,\Delta E/c^4$

Shrinking the observed region Δx increases the $\Delta E \sim R_g$, until

$$R_g(\Delta E) = \frac{2G\Delta E}{c^4} \to \Delta x \quad \Longrightarrow \quad (\Delta x)_g = \sqrt{\frac{G\hbar}{c^3}} = L_P$$

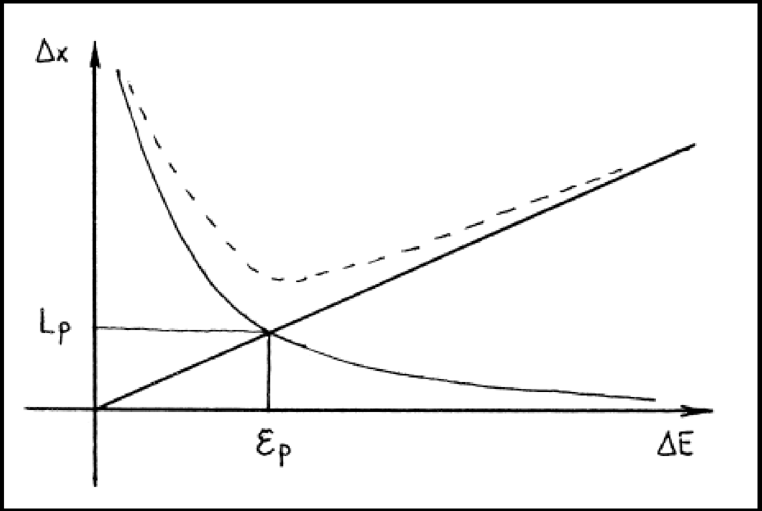
= **Planck length**. **A micro black hole originates**.



More refined details require larger energies, ⟹ expanding the R_g (ΔE), so hiding more details beyond the event horizon of the hole.

$$\Delta x \geq \begin{cases} \dfrac{\hbar c}{2\,\Delta E} & \text{for } \Delta E \leq \epsilon_P \\ \dfrac{2G\Delta E}{c^4} & \text{for } \Delta E > \epsilon_P \end{cases}$$

The situation can be described by the diagram, and by the inequality ($\Delta E = c\Delta p$)



$$\Delta x \geq \frac{\hbar}{2\Delta p} + \beta L_p^2 \frac{\Delta p}{\hbar}$$

with β = unknown parameter of order one.

ANALOGOUS ARGUMENT:

R.Adler, D.Santiago (1999), and R.Adler, P.Chen (2001) revisited Heisenberg microscope argument, but now including the gravitational effect of a single photon (in Newtonian Gravity). **Gravitational contribution to the uncertainty of particle position**:

$$\Delta x_g \approx Gh/\lambda c^3 \approx (G\hbar/c^3)/\lambda = l_p^2/\lambda$$

Photon partially transfers to the particle its momentum p = E/c = h/λ, then uncertainty in the particle momentum = Δp ~ p = h/λ. Total uncertainty:

$$\Delta x \sim \Delta x_Q + \Delta X_G \sim \frac{\hbar}{\Delta p} + \beta L_p^2 \frac{\Delta p}{\hbar}$$

Kempf, Mann, Brau, Vagenas, etc. (1995, 1999, 2008), translated GUP into a **deformed commutator**

$$[\hat{x}, \hat{p}] = i\hbar \left[1 + \beta \left(\frac{\hat{p}}{m_{\rm p}} \right)^2 \right]$$

and developed a Deformed Quantum Mechanics.

The dawn of Unruh Effect

"…an accelerated detector even in flat spacetime will detect particles in the vacuum… This result is exactly what one would expect of a detector immersed in a thermal bath of scalar photons of temperature $T_U = \frac{\hbar a}{2\pi c k_B}$"

(W.Unruh, PRD 1976).

Unruh Effect: heuristic derivation from HUP

Consider a particle of mass m at rest in an uniformly accelerated frame. The kinetic energy it acquires while the frame moves for a distance δx is

$$E_k = m\, a\, \delta x$$

Suppose this energy is just enough to create N pairs of the same kind of particles from the quantum vacuum (i.e. $E_k \sim 2Nm$, c=1), then the needed distance is $\delta x \sim 2N/a$

Particles and pairs so created are localized inside a space region of width δx, therefore the fluctuation in energy of each single particle is (HUP)

$$\delta E \simeq \frac{\hbar}{2\,\delta x} \simeq \frac{\hbar\, a}{4\, N}$$

If we interpret δE as a classical thermal agitation of the particles (i.e. $\frac{3}{2} k_B T \sim \delta E$), then

$$T = \frac{\hbar\, a}{6\, N\, k_{\mathrm{B}}} = T_{\mathrm{U}}$$

for $\mathrm{N} = \frac{\pi}{3} \sim 1$

Modified Unruh effect from GUP: heuristic derivation

We can repeat the same steps, using GUP instead of HUP

So we have
$$\delta x \simeq \frac{\hbar}{2\,\delta E} + 2\,\beta\,\ell_{\rm p}^2\,\frac{\delta E}{\hbar}$$

$$\delta x \simeq 2\,\frac{N}{a}, \;\; \delta E \simeq \frac{1}{2}\,k_{\rm B}\,T \quad \Longrightarrow \quad \frac{2\,\pi}{a} \simeq \frac{\hbar}{k_{\rm B}\,T} + \beta\,\ell_{\rm p}^2\,\frac{k_{\rm B}\,T}{\hbar}$$

Inverting the last for T=T(a), and expanding for $\beta\,k_{\rm B}\,T/m_{\rm p} \sim \beta\,m/m_{\rm p} \ll 1$ we get the GUP-modified Unruh temperature:

$$T \simeq T_{\rm U}\left(1 + \frac{\beta}{4}\,\frac{\ell_{\rm p}^2\,a^2}{\pi^2}\right) = T_{\rm U}\left[1 + \frac{\beta}{4}\left(\frac{k_{\rm B}\,T_{\rm U}}{m_{\rm p}}\right)^2\right]$$

Unruh effect from canonical QFT

Quantization of a scalar field in Minkowski space

Plane wave expansion

$$\phi(\mathbf{x}) = \int dk \left[a_k U_k(\mathbf{x}) + a_k^\dagger U_k^*(\mathbf{x})\right]$$

$$[a_k, a_{k'}^\dagger] = \delta(k - k')$$

$$a_k |0\rangle_{\mathrm{M}} = 0 \quad \forall k$$

$$U_k(\mathbf{x}) = (4\pi\omega_k)^{-\frac{1}{2}} e^{i(k\,x - \omega_k\,t)}$$

$$\omega_k = \sqrt{m^2 + k^2}$$

Lorentz-boost mode expansion (Takagi 1986)

$$\widetilde{U}_\Omega^{(\sigma)}(\mathbf{x}) = \int dk\, p_\Omega^{(\sigma)*}(k)\, U_k(\mathbf{x})$$

$$p_\Omega^{(\sigma)}(k) = \frac{1}{\sqrt{2\pi\,\omega_k}} \left(\frac{\omega_k + k}{\omega_k - k}\right)^{i\sigma\Omega/2}$$

$$\phi(\mathbf{x}) = \int_0^{+\infty} d\Omega \sum_{\sigma=\pm} \left[d_\Omega^{(\sigma)} \widetilde{U}_\Omega^{(\sigma)}(\mathbf{x}) + d_\Omega^{(\sigma)\dagger} \widetilde{U}_\Omega^{(\sigma)*}(\mathbf{x})\right]$$

Link boost-mode operators with Minkowski annihilators

$$d_\Omega^{(\sigma)} = \int dk\, p_\Omega^{(\sigma)}(k)\, a_k$$

In these operators the Lorentz boost generator is diagonal

$$M^{(1,0)} = \int d^3\kappa \sum_\sigma \sigma\Omega \left(d_\kappa^{(\sigma)\dagger} d_\kappa^{(\sigma)} + \bar{d}_\kappa^{(\sigma)\dagger} \bar{d}_\kappa^{(\sigma)}\right)$$

Canonical commutators

$$\left[a_k, a_{k'}^\dagger\right] = \delta(k - k') \Longleftrightarrow \left[d_\Omega^{(\sigma)}, d_{\Omega'}^{(\sigma')\dagger}\right] = \delta_{\sigma\sigma'}\delta(\Omega - \Omega')$$

Therefore also $d_\Omega^{(\sigma)}$ are annihilation operators of Minkowski quanta

$$d_\kappa^{(\sigma)}|0\rangle_M = 0$$

The two different field-expansions are equivalent for inertial observers in Minkowski spacetime.

Rindler – Fulling – Unruh quantization in uniformly accelerating frame. Rindler space:

- **Rindler coordinates**

$$t = \xi \sinh \eta, \quad x = \xi \cosh \eta$$

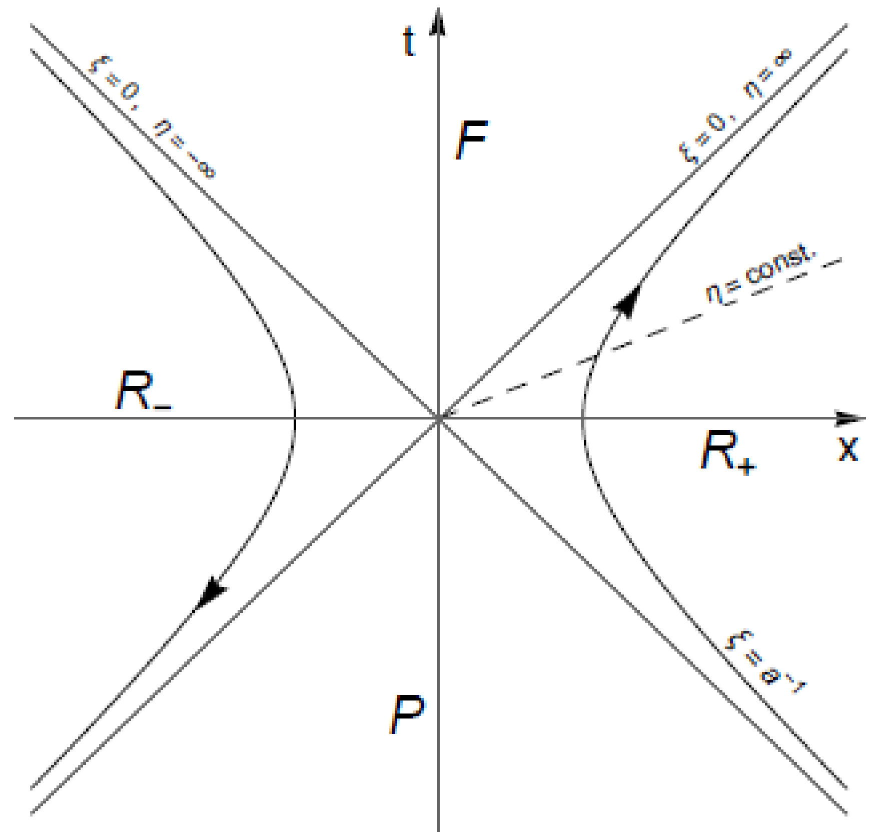
- Rindler vs Minkowski metrics

$$ds^2_{\mathrm{M}} = (dt)^2 - (dx)^2 \rightarrow$$

$$ds^2_{\mathrm{R}} = \xi^2 d\eta^2 - d\xi^2$$

- Worldline of a Rindler observer

$$\eta = a\tau, \quad \xi = \mathrm{const} \equiv a^{-1}$$



For c≠1, $\tau=\eta c/a$, $\eta=a\tau/c$ (dimensionless)

Worldline of uniformly accelerated Rindler observer $\xi(\tau) = c^2/a$

Quantization of the scalar field in Rindler space

$$\phi(\mathbf{x}) = \int_0^{+\infty} d\Omega \sum_{\sigma=\pm} \left[b_\Omega^{(\sigma)}\, u_\Omega^{(\sigma)}(\mathbf{x}) + b_\Omega^{(\sigma)\dagger}\, u_\Omega^{(\sigma)*}(\mathbf{x}) \right]$$

where the positive frequency solutions of the Klein-Gordon equation in Rindler coordinates is $u_\Omega^{(\sigma)}(\mathbf{x}) = N_\Omega\, \theta(\sigma\,\xi)\, K_{i\,\Omega}^{(\sigma)}(m\,\xi)\, e^{-i\,\sigma\,\Omega\,\eta}$ (Takagi 1986)

with: proper frequency ω measured by a Rindler observer is linked to Ω by (c≠1)

$$\omega\,\tau = \omega(\eta c/a) = (\omega c/a)\eta \equiv \Omega\,\eta$$

$\omega = a\Omega/c$, $\sigma = \pm$ refers to the right/left wedges $R_\pm$

$K_{i\Omega}$ = modified Bessel function of the second kind, N_Ω = normalization factor.

The ladder operators satisfy the canonical commutation relations

$$\left[b_\Omega^{(\sigma)}, b_{\Omega'}^{(\sigma')\dagger} \right] = \delta_{\sigma\sigma'}\,\delta(\Omega - \Omega')$$

The Rindler vacuum is accordingly defined as $b_\Omega^{(\sigma)}|0\rangle_{\rm R} = 0$ for all σ and Ω.

Connection between inertial and accelerated observers

Minkowski (boost-mode) quantization:

$$\phi = \int d\Omega \sum_{\sigma=\pm} \left[d_\Omega^{(\sigma)} \, \tilde{U}_\kappa^{(\sigma)} + d_\Omega^{(\sigma)\dagger} \, \tilde{U}_\kappa^{(\sigma)*} \right],$$

Rindler quantization:

$$\phi = \int d\Omega \sum_{\sigma=\pm} \left[b_\Omega^{(\sigma)} \, u_\Omega^{(\sigma)} + b_\Omega^{(\sigma)\dagger} \, u_\Omega^{(\sigma)*} \right]$$

↘ ↙

Bogoliubov transformation

$$b_\Omega^{(\sigma)} = [1 + \mathcal{N}(\Omega)]^{1/2} \, d_\Omega^{(\sigma)} + \mathcal{N}(\Omega)^{1/2} \, d_\Omega^{(-\sigma)\dagger}$$

with B.E. distribution

$$\mathcal{N}(\Omega) = \frac{1}{e^{2\pi\Omega} - 1}$$

Spectrum of Rindler quanta in the Minkowski vacuum:

$$\langle 0_M | \, b_\Omega^{(\sigma)\dagger} \, b_{\Omega'}^{(\sigma')} \, | 0_M \rangle = \mathcal{N}(\Omega) \, \delta_{\sigma\sigma'} \, \delta(\Omega - \Omega')$$

Uniformly accelerated observer perceives Minkowski vacuum as a **thermal bath** of Rindler quanta with a **temperature proportional to the acceleration**.

UNRUH TEMPERATURE

$$2\pi\Omega = \frac{2\pi}{a} a\,\Omega = \frac{\hbar\, a\, \Omega}{k_B T_U} = \frac{\hbar\,\omega}{k_B T_U}$$

$$T_U = \frac{a}{2\pi} = \frac{\hbar a}{2\pi c k_B}$$

Modified Unruh effect from GUP: QFT derivation

GUP and one-dimensional quantum harmonic oscillator

$$A = \frac{1}{\sqrt{2\, m\, \hbar\, \omega}}\left(m\omega\hat{x} + i\,\hat{p}\right)$$

$$A^\dagger = \frac{1}{\sqrt{2\, m\, \hbar\, \omega}}\left(m\omega\hat{x} - i\,\hat{p}\right)$$

$$\hat{x} = \sqrt{\frac{\hbar}{2\, m\, \omega}}(A^\dagger + A)$$

$$\hat{p} = i\sqrt{\frac{m\,\hbar\,\omega}{2}}(A^\dagger - A)$$

Then $[A, A^\dagger] = \frac{1}{i\hbar}[\hat{x}, \hat{p}]$ and from $[\hat{x}, \hat{p}] = i\hbar\left[1 + \beta\left(\frac{\hat{p}}{m_{\rm p}}\right)^2\right]$

we get the deformed algebra of the one dimensional harmonic oscillator

$$[A, A^\dagger] = \frac{1}{1-\alpha}[1 - \alpha(A^\dagger A^\dagger + A\, A - 2\, A^\dagger A)]$$

with

$$\alpha = \beta\,\frac{m\,\hbar\,\omega}{2\, m_{\rm p}^2}$$

* Deformed commutator for scalar field in plane-wave representation

Since for a given momentum k, the energy $\hbar\omega_k$ of the scalar field plays the role of the mass ***m*** of the harmonic oscillator, **then**

$$\alpha = \beta \frac{m\,\hbar\,\omega}{2\,m_{\rm p}^2} \quad \Rightarrow \quad \tilde{\alpha} = \beta \frac{\hbar^2 \omega_k^2}{2\,m_{\rm p}^2} = 2\,\beta\,\ell_{\rm p}^2\,\omega_k^2$$

And the deformed commutator becomes:

$$[A_k, A_{k'}^\dagger] = \frac{1}{1-\tilde{\alpha}}\left[1 - \tilde{\alpha}\left(A_k^\dagger A_{k'}^\dagger + A_k A_{k'} - 2\,A_k^\dagger A_{k'}\right)\right]\delta(k-k')$$

* Deformed commutator for scalar field in boost-mode representation

In the limit of small deformation ($\beta p^2/\, m_p^2 \ll 1$), is reasonable to assume the same structure for the deformed algebra of the boost modes

$$\left[D_\Omega^{(\sigma)}, D_{\Omega'}^{(\sigma')\dagger}\right] = \frac{1}{1-\gamma}\left[1 - \gamma\left(D_\Omega^{(\sigma)\dagger} D_{\Omega'}^{(-\sigma')\dagger} + D_\Omega^{(\sigma)} D_{\Omega'}^{(-\sigma')} - D_\Omega^{(\sigma)\dagger} D_{\Omega'}^{(\sigma')} - D_\Omega^{(-\sigma)\dagger} D_{\Omega'}^{(-\sigma')}\right)\right]\delta_{\sigma\sigma'}\,\delta(\Omega-\Omega')$$

where now

$$\gamma = \beta \frac{\hbar^2 \omega^2}{2\,m_{\rm p}^2} = \beta \frac{\hbar^2 a^2\, \Omega^2}{2\,m_{\rm p}^2} = 2\,\beta\,\ell_{\rm p}^2\, a^2\, \Omega^2$$

being $\omega = a\Omega$ (c=1) the Rindler proper frequency

Note that

- The deforming parameter $\alpha\sim \longrightarrow \gamma$ has been defined to suit the boost mode representation

  (i.e. plane frequency $\omega_k \rightarrow \omega = a\Omega$ boost mode frequency)
- The D-commutator has been defined so that the ladder operators D in the wedges R+, R- are still commuting with each other.

The D-commutator has been symmetrized with respect to $\pm\sigma$ so that

$$\left[D_{\Omega}^{(\sigma)}, D_{\Omega'}^{(\sigma')\dagger}\right] = \left[D_{\Omega}^{(-\sigma)}, D_{\Omega'}^{(-\sigma')\dagger}\right]$$

The deformation of the D-algebra leads to an analogous modification of the commutator of the Rindler B-operators.

The Bogoliubov transformation between B and D is now

$$B_{\Omega}^{(\sigma)} = [1+\mathcal{N}(\Omega)]^{1/2} D_{\Omega}^{(\sigma)} + \mathcal{N}(\Omega)^{1/2} D_{\Omega}^{(-\sigma)\dagger}$$

GUP effect on the Unruh temperature

Distribution of B-quanta in the Minkowski vacuum

$$\langle 0_M | B_{\Omega}^{(\sigma)\dagger} B_{\Omega'}^{(\sigma')} | 0_M \rangle = \frac{1}{\left(e^{2\pi\Omega} - 1\right)\left(1-\gamma\right)} \delta_{\sigma\sigma'} \delta(\Omega - \Omega') \simeq \frac{1}{e^{2\pi\Omega - \gamma} - 1} \delta_{\sigma\sigma'} \delta(\Omega - \Omega')$$

This can be interpreted as a B-E thermal distribution with a

SHIFTED UNRUH TEMPERATURE such that

$$2\pi\Omega - \gamma = \frac{\hbar\, a\, \Omega}{k_B T_U} - \gamma \equiv \frac{\hbar\, a\, \Omega}{k_B T}$$

$$T = \frac{T_U}{1 - \beta\pi\Omega k_B^2 T_U^2 / m_p^2} \simeq T_U \left(1 + \beta\pi\Omega \left(\frac{k_B T_U}{m_p}\right)^2\right) = T_U \left(1 + \beta\pi\Omega \frac{\ell_p^2 a^2}{\pi^2}\right)$$

Remark: T contains an **explicit dependence** on the Rindler frequency $\Omega = \omega / a$

Expected, since the fundamental commutator depends on p^2 , i.e. on the energy of the considered quantum mode.

Thermodynamic argument (to get rid of Ω):

Small deformations of HUP $\Longrightarrow$ The modified Unruh radiation is still close to thermal black body spectrum $\Longrightarrow$ The majority of Unruh quanta are emitted around the Rindler frequency ω such that $\hbar\omega = k_B T_U \Longrightarrow \Omega \approx 1/2\pi$. For this typical frequency

$$T \simeq T_{\mathrm{U}}\left[1+\frac{\beta}{2}\left(\frac{k_{\mathrm{B}}T_{\mathrm{U}}}{m_{\mathrm{p}}}\right)^2\right] = T_{\mathrm{U}}\left(1+\frac{\beta}{2}\frac{\ell_{\mathrm{p}}^2 a^2}{\pi^2}\right)$$

...which equals, almost numerically, with the heuristic result

$$T \simeq T_{\mathrm{U}}\left(1+\frac{\beta}{4}\frac{\ell_{\mathrm{p}}^2 a^2}{\pi^2}\right) = T_{\mathrm{U}}\left[1+\frac{\beta}{4}\left(\frac{k_{\mathrm{B}}T_{\mathrm{U}}}{m_{\mathrm{p}}}\right)^2\right]$$

Conclusions and outlook

We investigated

- Deviation from thermality of Unruh radiation in the context of GUP
- Small deformations of HUP $\Longrightarrow$ The resulting Unruh distribution still exhibits a thermal spectrum with a modified temperature

$$T \simeq T_{\mathrm{U}}\left(1 + \beta\, \mathcal{O}(a^2)\right)$$

- Good agreement between the heuristic and the field theoretical approaches

- To be investigated: What happens beyond the approximation of equal deformed algebras for the A- and D- operators? (Deformation of algebra for field operators)