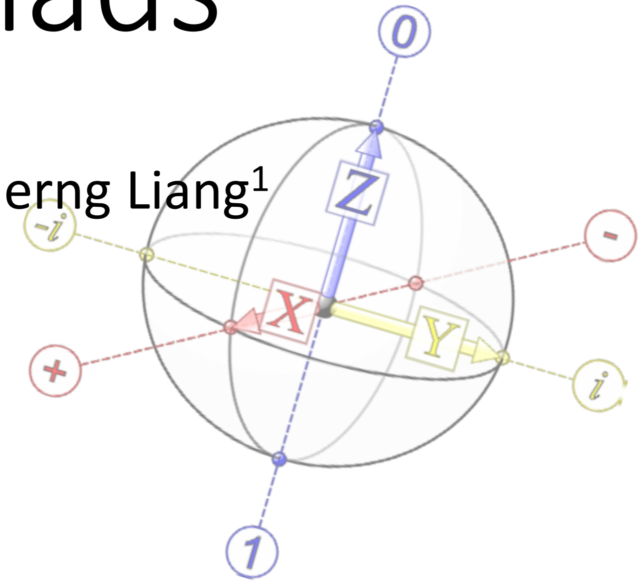
Multipartite Bell-inequality violation using randomly chosen triads



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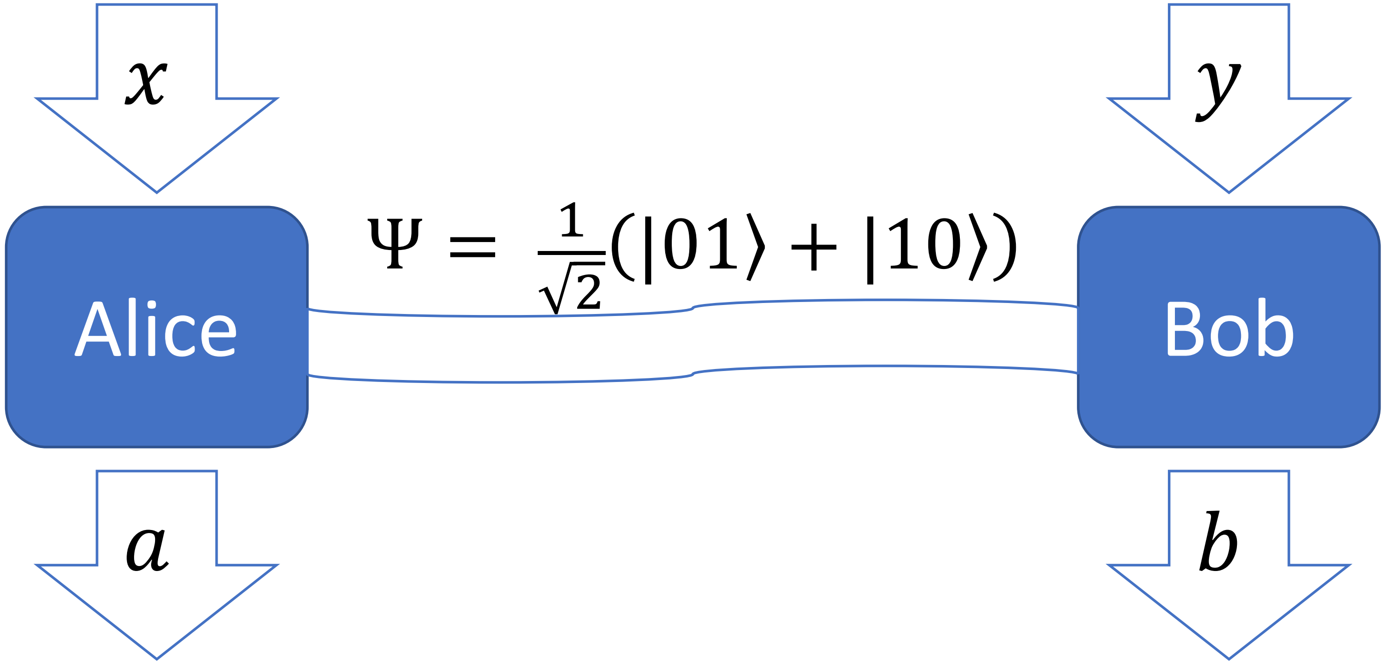
Pic from: https://openclipart.org/detail/142879/qubit-bloch-sphere

Shared Reference Frame in a Bell Scenario

$$\frac{1}{2}[\langle A_1B_1\rangle + \langle A_2B_1\rangle + \langle A_1B_2\rangle - \langle A_2B_2\rangle] \leq 1 \quad \sqrt{2}$$

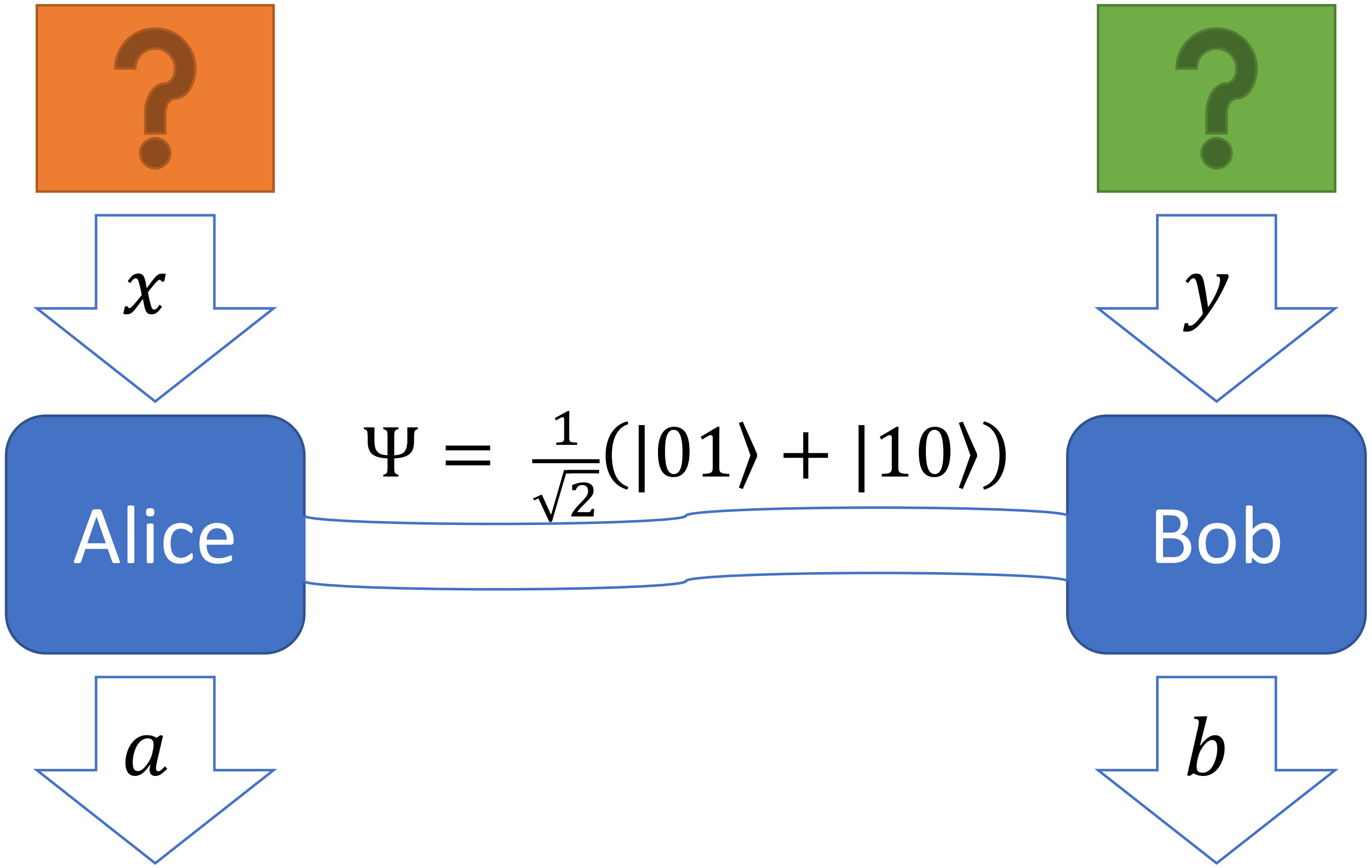
$A_1 = \sigma_z, A_2 = \sigma_x$ $\quad A_1, A_2$

$\boldsymbol{B_1} = \frac{\boldsymbol{\sigma'_z} + \sigma'_x}{\sqrt{2}}, B_2 = \frac{\sigma_z \sigma'_z - \sigma_x \sigma'_x}{\sqrt{2}\sqrt{2}}$ $\quad B_1, B_2$

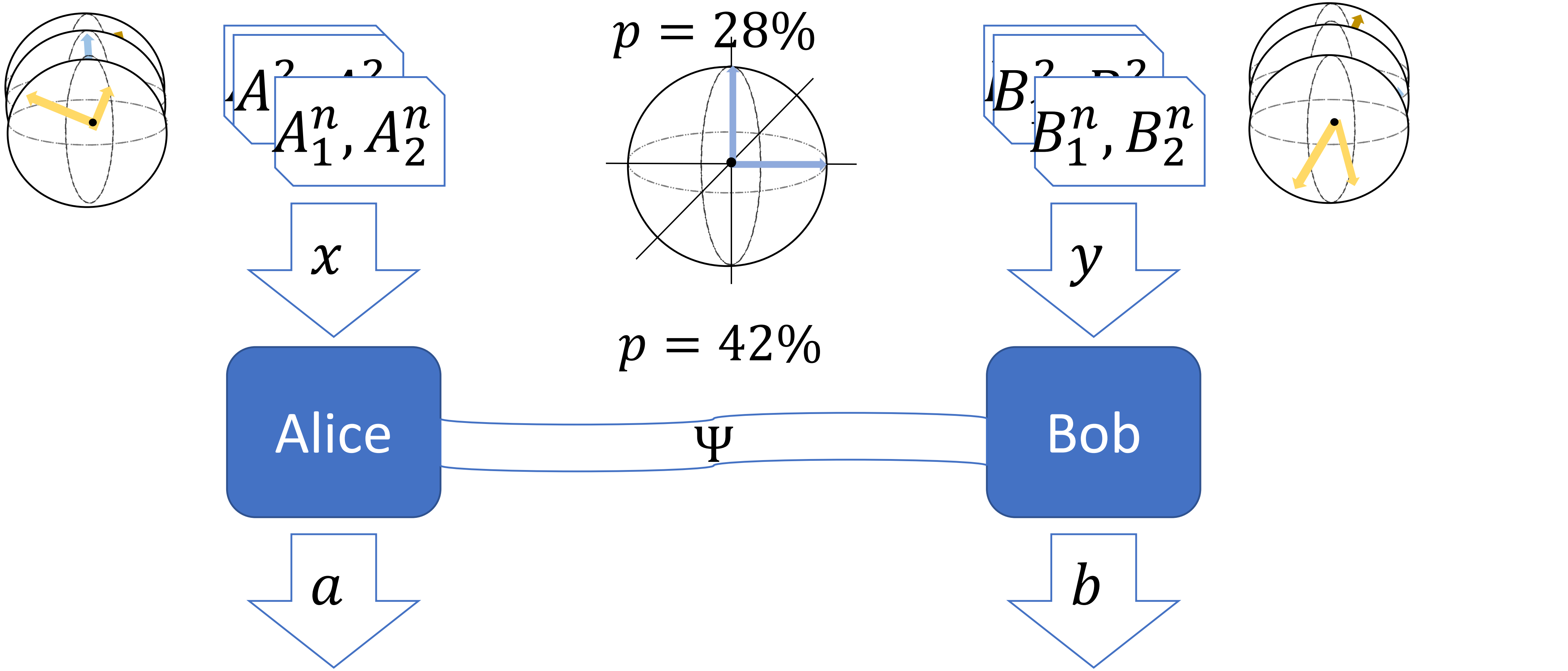


Random Measurement in a Bell Scenario

$$\frac{1}{2}[\langle A_1 B_1\rangle + \langle A_2 B_1\rangle + \langle A_1 B_2\rangle - \langle A_2 B_2\rangle] = ?$$

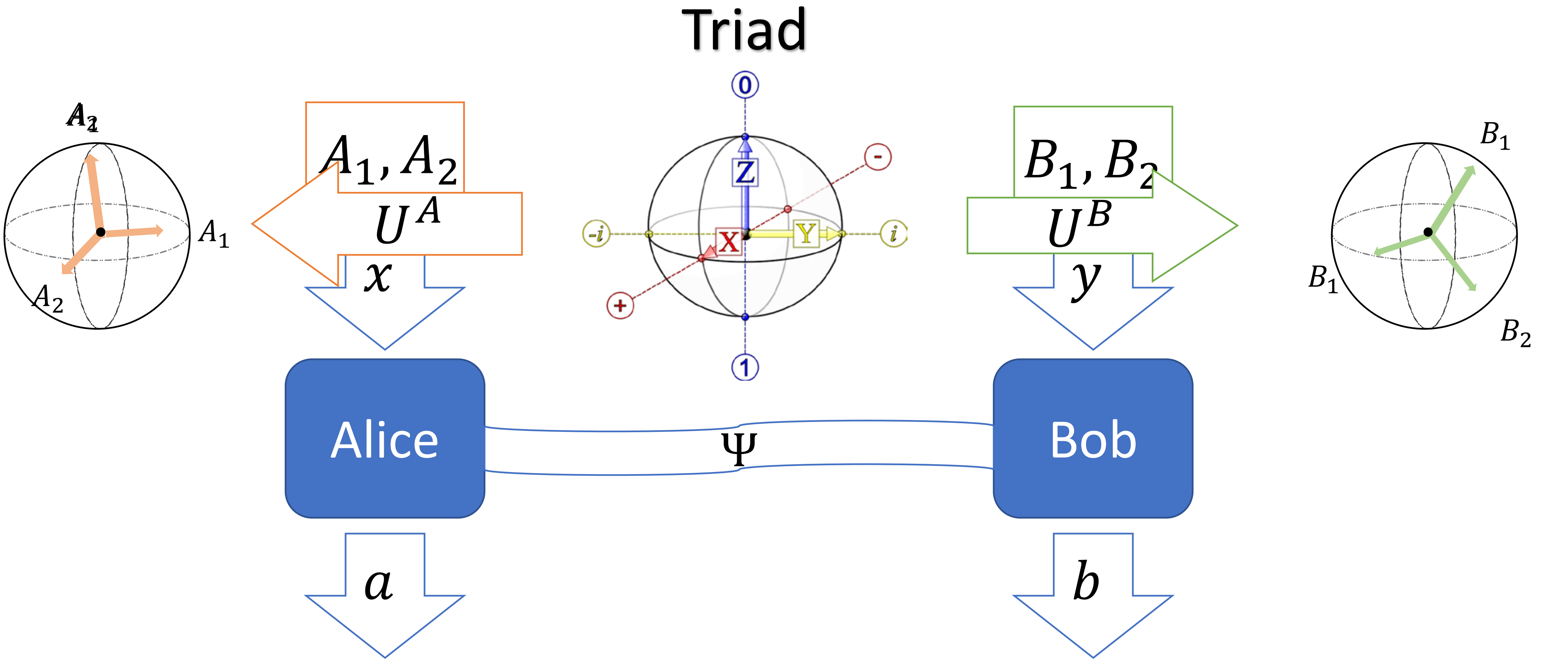


Probability of Violation $\sim \frac{\text{num.of simulations with violation}}{n}$



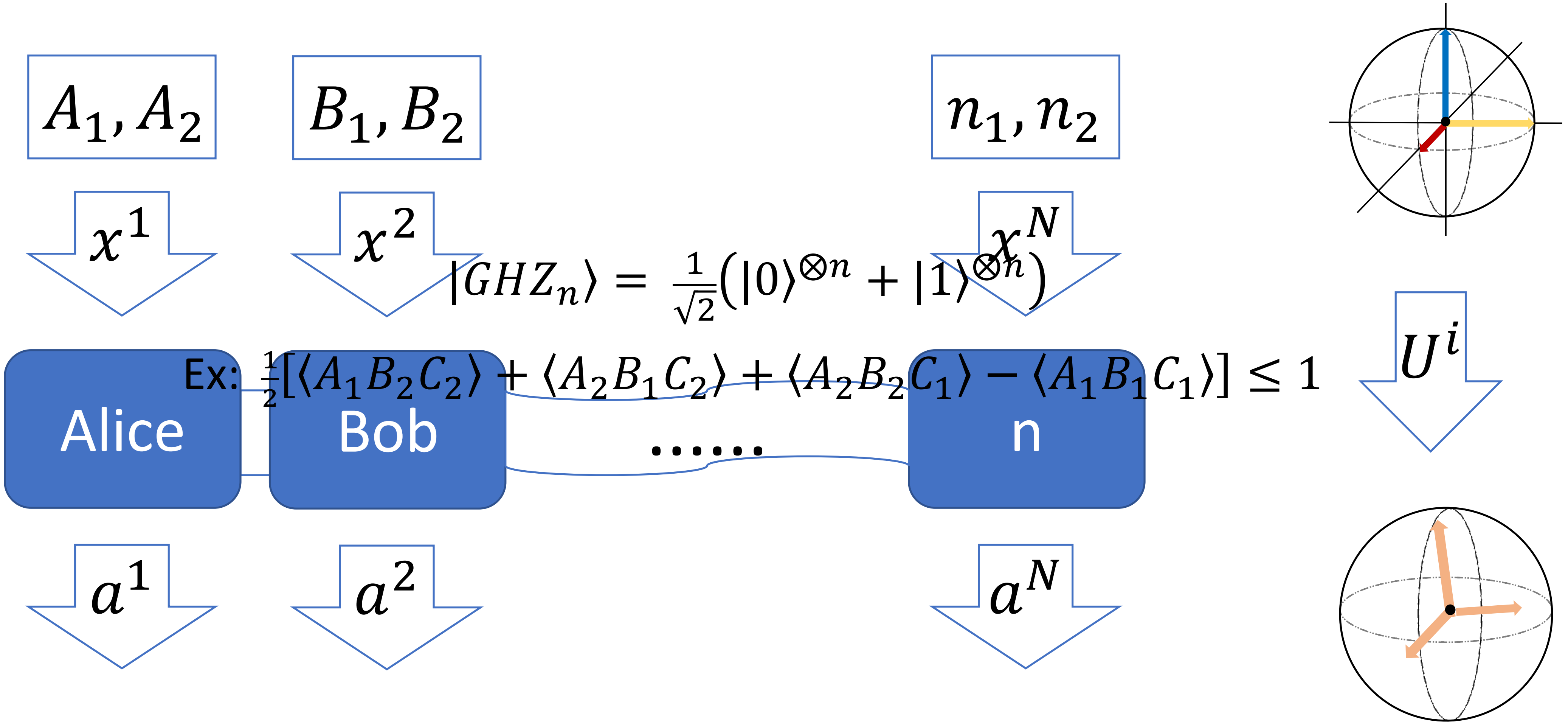
Liang *et al*. (2010). Phys. Rev. Lett. **104**, 050401.

Improve the Chance Using Triad



Shadbolt *et al*. (2012). Scientific Reports. 2. 470. 10.1038/srep00470.

Generalizing to n Parties

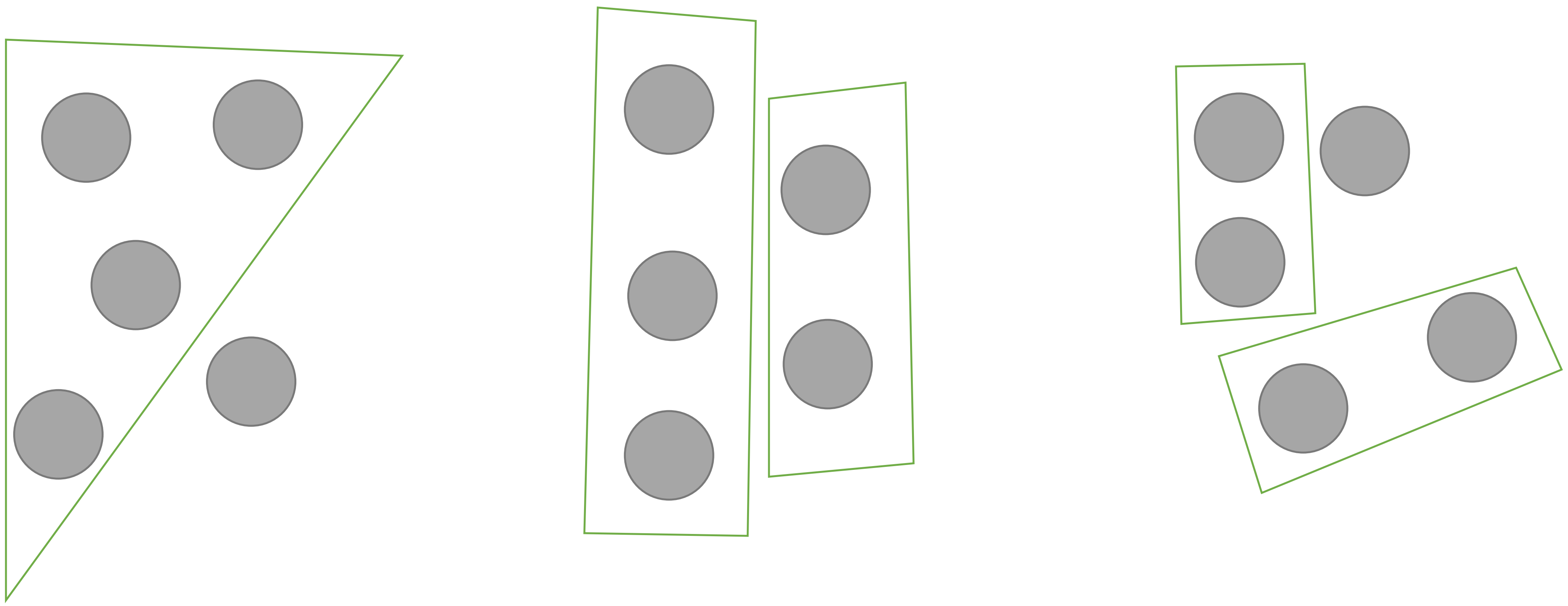


$$|GHZ_n\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle^{\otimes n} + |1\rangle^{\otimes n}\right)$$

Ex: $\frac{1}{2}[\langle A_1 B_2 C_2\rangle + \langle A_2 B_1 C_2\rangle + \langle A_2 B_2 C_1\rangle - \langle A_1 B_1 C_1\rangle] \leq 1$

k-Producibility

Entanglement depth of k: k-producible but not $(k\text{-}1)$-producible



Probability of Certifying Entanglement Depth

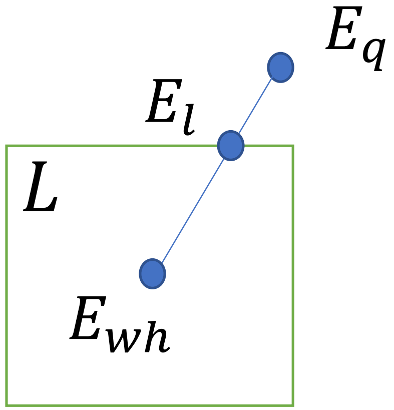
n	3	4	5	6	7	8
ED /#	4×10^6	5×10^6	2×10^6	4×10^5	4×10^5	2×10^5
2	99.99%	100%	100%	100%	100%	100%
3	45.89%	99.11%	100%	100%	100%	100%
4	–	22.54%	89.84%	–	99.26%	99.99%
5	–	–	8.83%	70.94%	–	–
6	–	–	–	2.86%	47.84%	–
7	–	–	–	–	0.82%	27.70%
8	–	–	–	–	–	0.22%

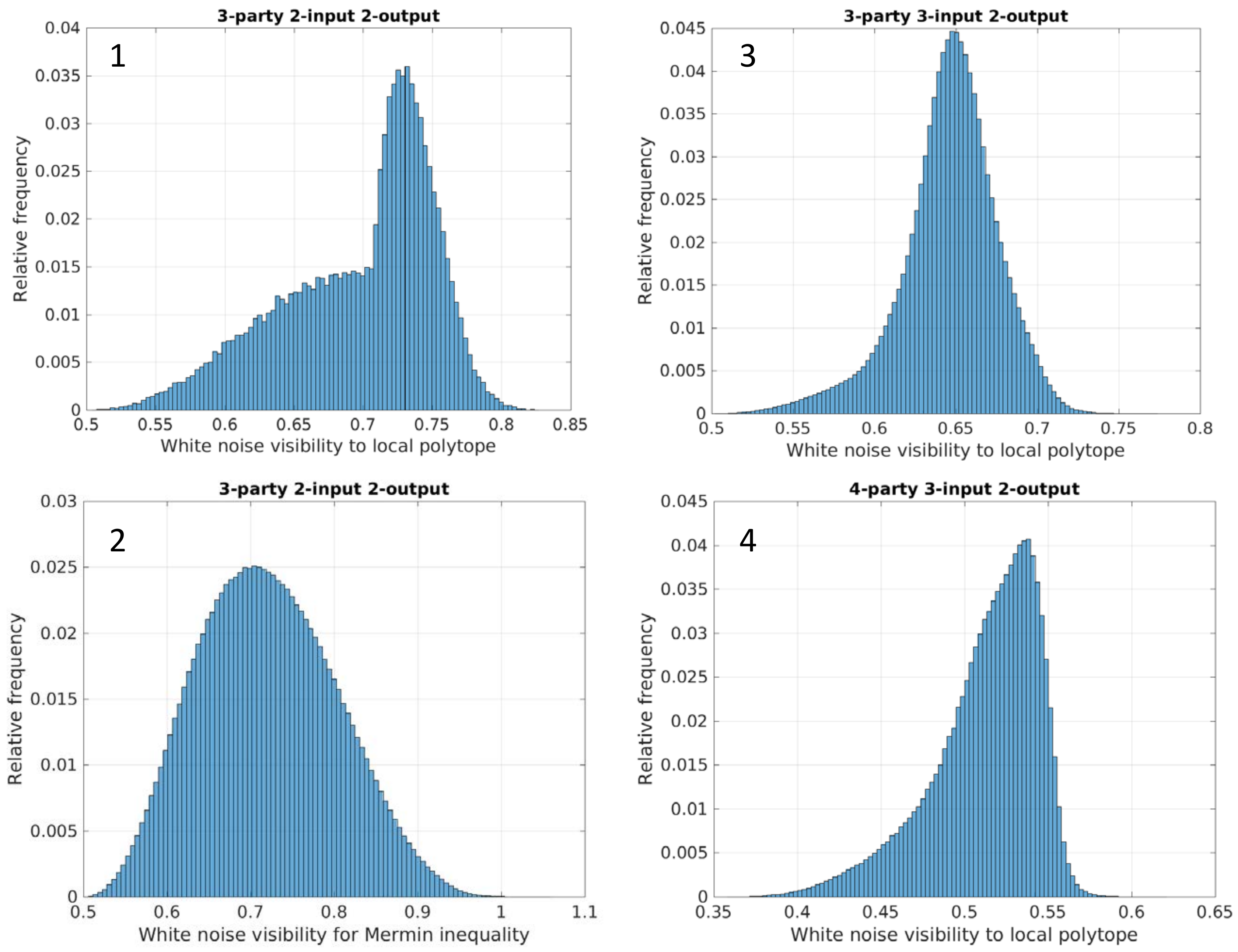
k	3	4	5	6	7	8
1	1	1	1	1	1	1
2	$\sqrt{2}$	$\sqrt{2}$	$\sqrt{2}$	$\sqrt{2}$	$\sqrt{2}$	$\sqrt{2}$
3	–	2	2	$2\sqrt{2}$	$2\sqrt{2}$	$2\sqrt{2}$
4	–	–	$2\sqrt{2}$	$2\sqrt{2}$	4	$4\sqrt{2}$
5	–	–	–	4	4	$4\sqrt{2}$
6	–	–	–	–	$4\sqrt{2}$	$4\sqrt{2}$
7	–	–	–	–	–	8

Visibility

$$E_l = vE_q + (1 - v)E_{wh}$$

E_q: quantum point,
E_{wh}: white noise,
v: visibility,
E_l: local point





3-party 2-input 2-output
1
Relative frequency
White noise visibility to local polytope
0.04
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
0.5
0.55
0.6
0.65
0.7
0.75
0.8
0.85
3-party 3-input 2-output
3
Relative frequency
White noise visibility to local polytope
0.045
0.04
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
0.5
0.55
0.6
0.65
0.7
0.75
0.8
3-party 2-input 2-output
2
Relative frequency
White noise visibility for Mermin inequality
0.03
0.025
0.02
0.015
0.01
0.005
0
0.5
0.6
0.7
0.8
0.9
1
1.1
4-party 3-input 2-output
4
Relative frequency
White noise visibility to local polytope
0.045
0.04
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
0.35
0.4
0.45
0.5
0.55
0.6
0.65

Other Bell Inequalities in Tripartite Scenario

ED / F	2	7	8	22	24	26	27	33	39	40	42
2	99.99	55.84	100	100	100	96.36	99.99	100	100	99.67	100
3	45.83	6.82	4.97	0.18	16.63	4.00	0.65	62.02	39.32	2.28	3.08

Family 33:

$$0 \leq 6 - E(A_1) - E(A_2) - E(B_1) + E(A_2B_1) - E(B_2) + E(A_1B_2) - E(C_1) + E(A_2C_1) - 2E(A_2B_1C_1) + E(B_2C_1) - 2E(A_1B_2C_1) - E(A_2B_2C_1) - E(C_2) + E(A_1C_2) + E(B_1C_2) - 2E(A_1B_1C_2) - E(A_2B_1C_2) - E(A_1B_2C_2) + 3E(A_2B_2C_2)$$

C. Śliwa, Phys. Lett. A, vol. 317, no. 3, pp. 165 – 168 (2003).

Non overlapping k-producible Bounds

k	I_{S7_n}	I_{FG_4}	I_{FG_5}	I_{FG_6}
1	1	3	4	5
2	$\sqrt{2}$	3.6742	4.6188	5.5902
3	5/3	4.4037	5.1962	6.0977
4	1.8482	5	5.4037	6.1962
5	1.9746	–	6	6.4037
6	2.0777	–	–	7

ED	I_{S7_4}	I_{FG_4}	I_{M_4}
2	100	91.13	100
3	97.39	32.48	99.11
4	27.29	1.30	22.54

$$I_{S7_n} = 2^{1-n}\Sigma_{\vec{x}\in\{1,0\}^n} E_n(\vec{x}) - E_n(\vec{1}_n)$$

$$I_{FG_n} = E_n(1,2,\dots,2) + \circlearrowright' - E_n(\vec{1}_3, \vec{2}_{n-3})$$

I_{S7_n}: Y.-C. Liang *et al.*, Phys. Rev. L, vol. 114, p. 190401 (2015).
I_{FG_n}: P.-S. Lin *et al.*, arXiv, no. 1903.02171, 2019.

Summary

- Correlations from random triad (optimized over all choices of 2 out of 3 measurements per party) will always violate a Bell inequality.
- Even randomly generated correlation features rather robust resistance to white noise
- Probability of certifying > 2-party entanglement is ~ 100% for $N>2$
- But probability of certifying genuine N-party entanglement decreases.