

SUFFICIENT CRITERION FOR THE UNSTEERABILITY OF TWO-QUBIT STATES

Joseph Bowles

Flavien Hirsch, Marco Quintino, Nicolas Brunner

$$a = -1$$

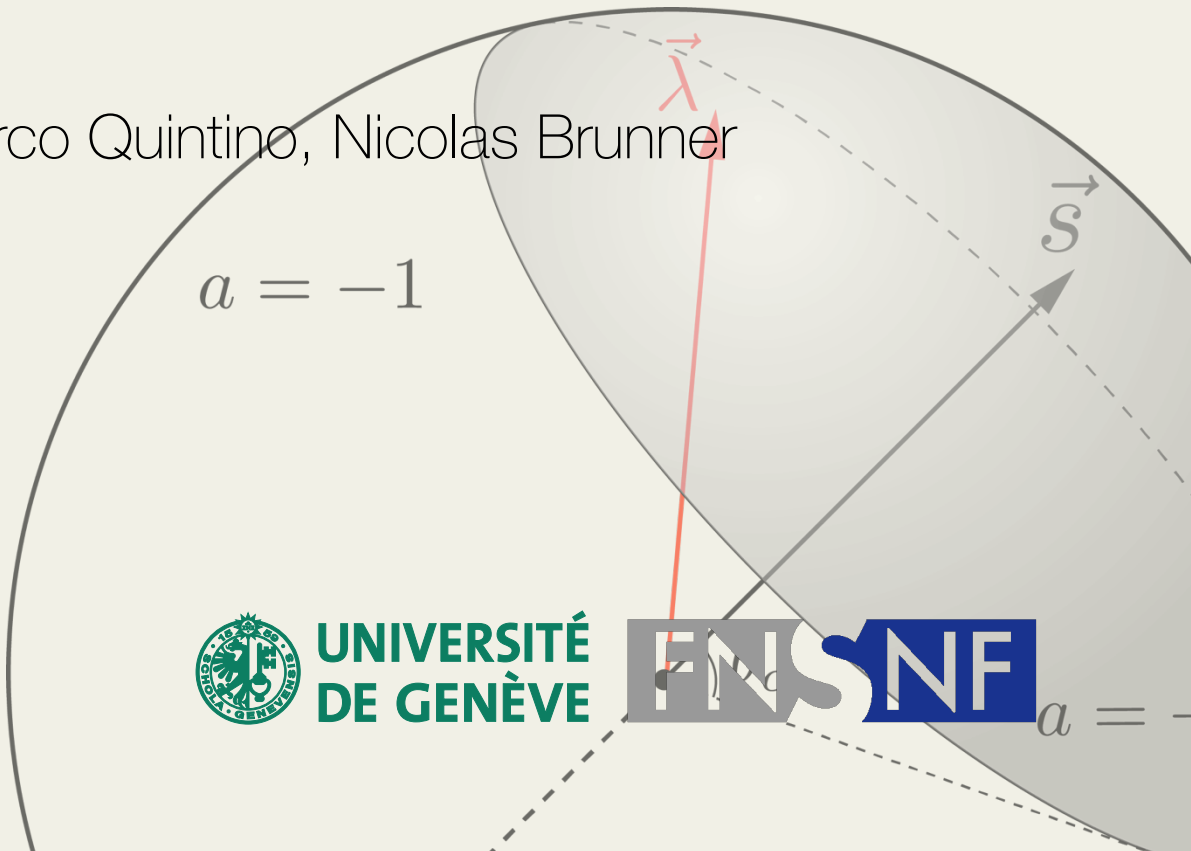
arXiv:1510:06721



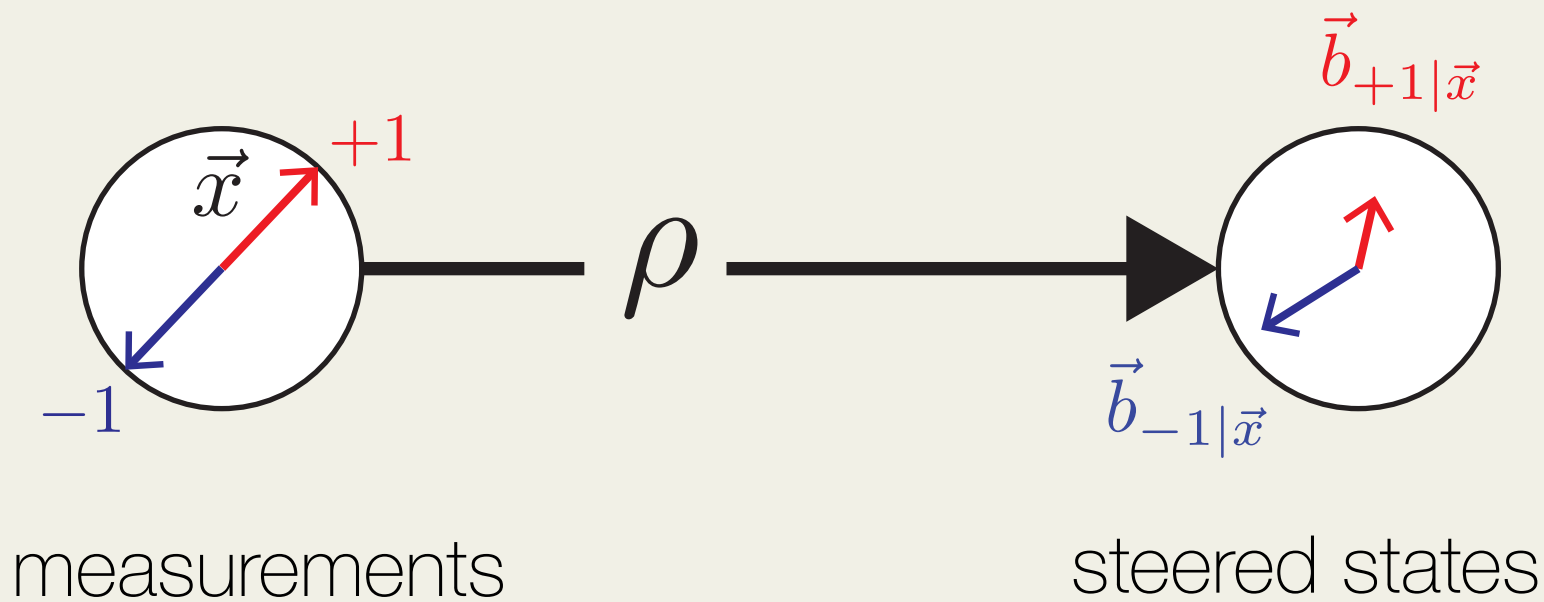
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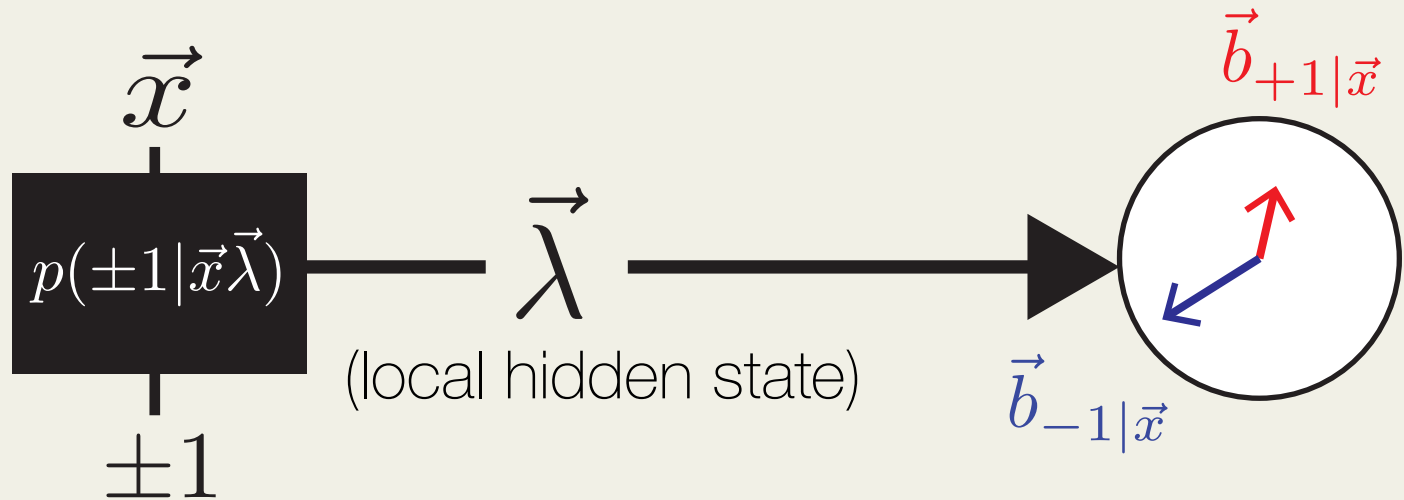
$a = -$



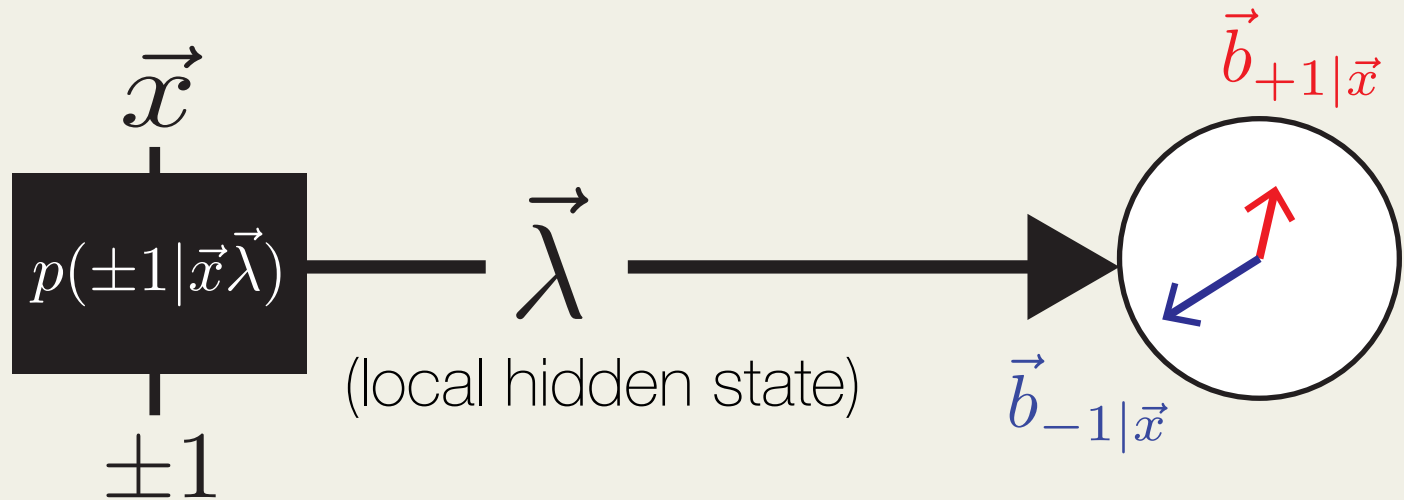
Steering



Local Hidden State Models

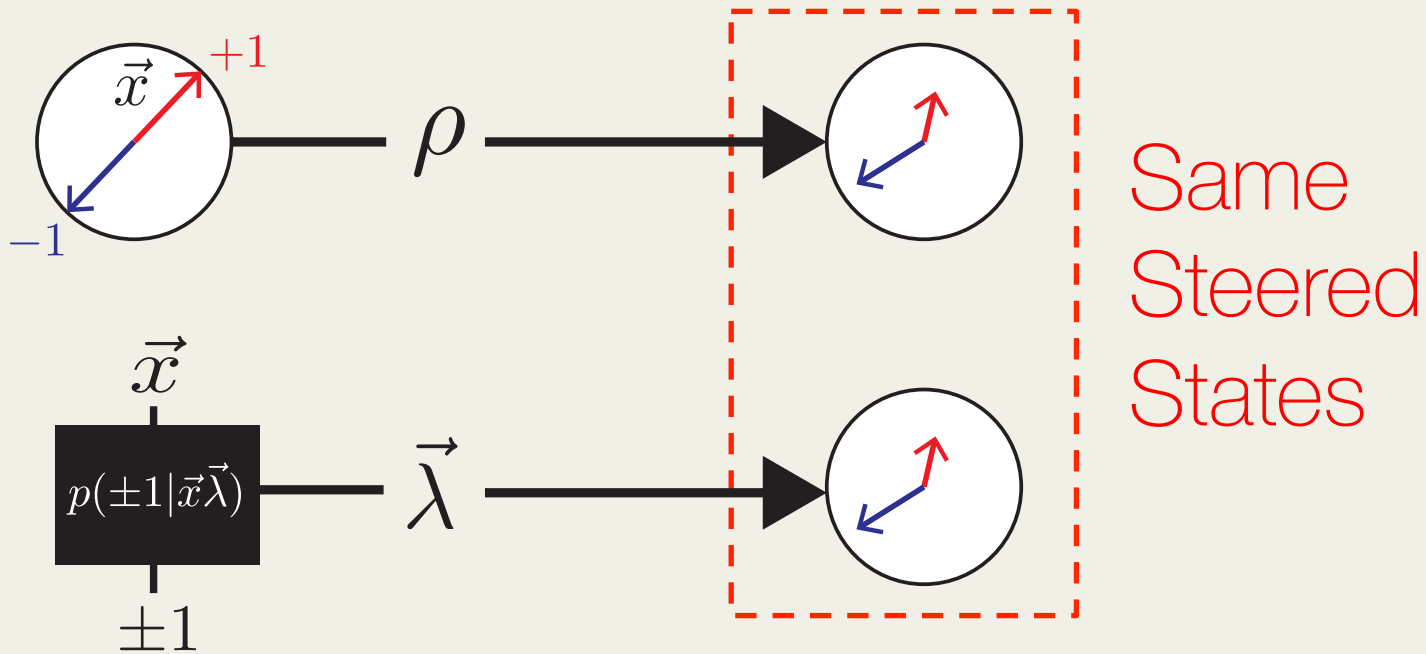


Local Hidden State Models



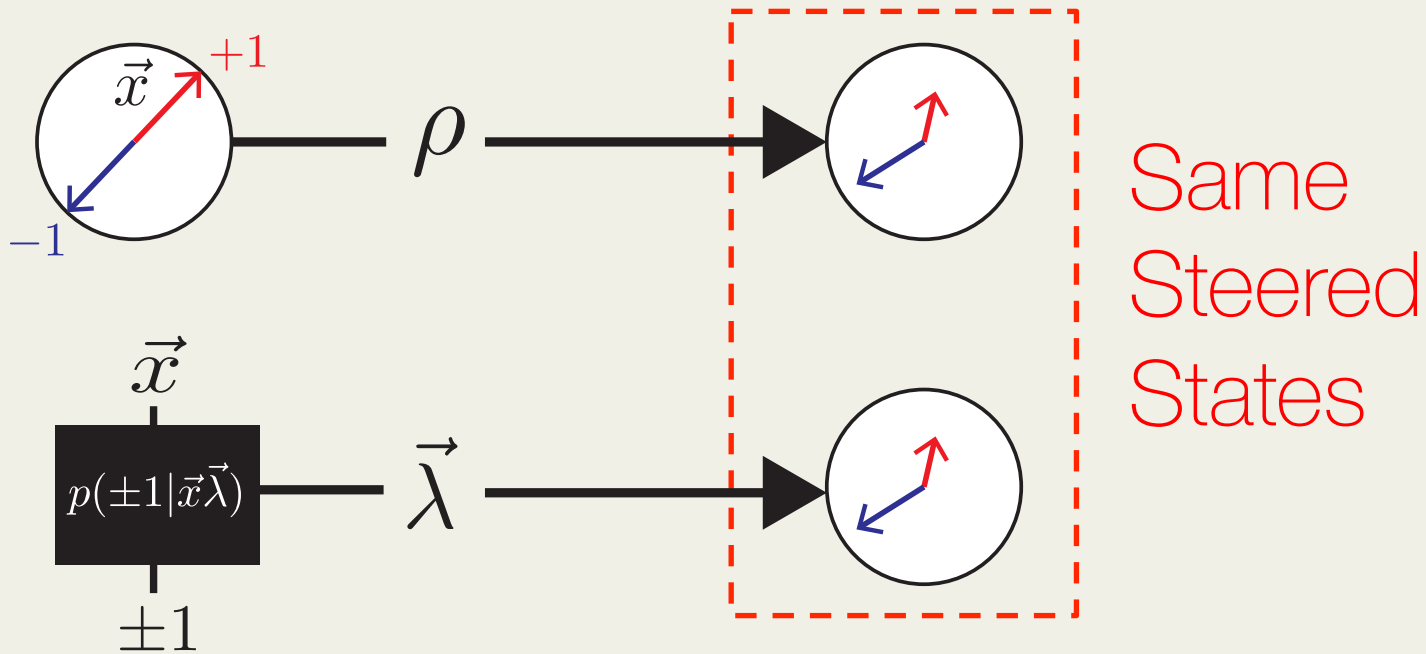
$$\vec{b}_{\pm 1 | \vec{x}} = \int q_{\vec{\lambda}} p(\pm 1 | \vec{x} \vec{\lambda}) \vec{\lambda} d\vec{\lambda}$$

Unsteerable states



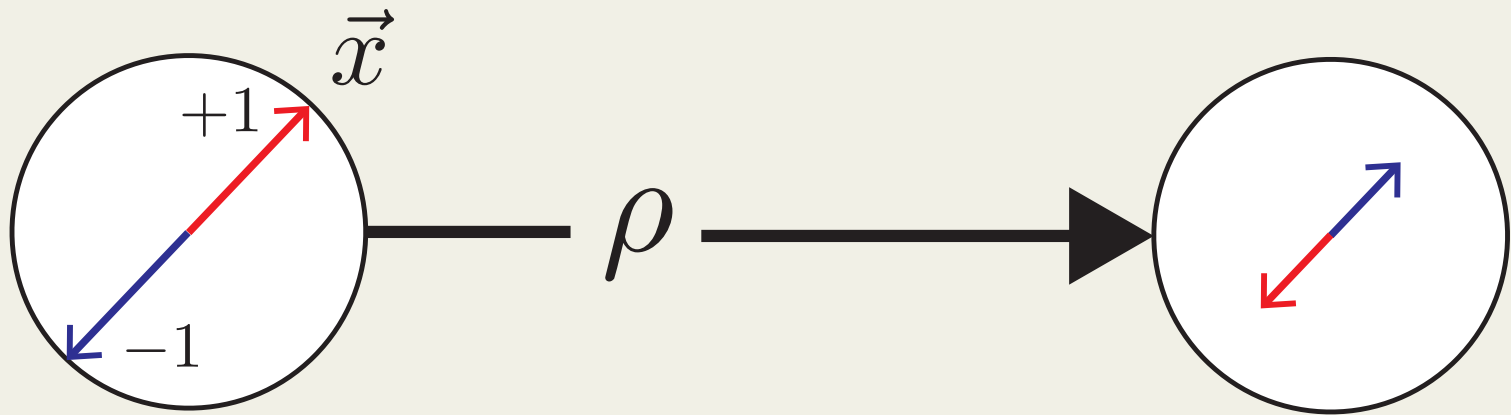
LHS model $\iff \rho$ unsteerable
for all \vec{x}

Unsteerable states



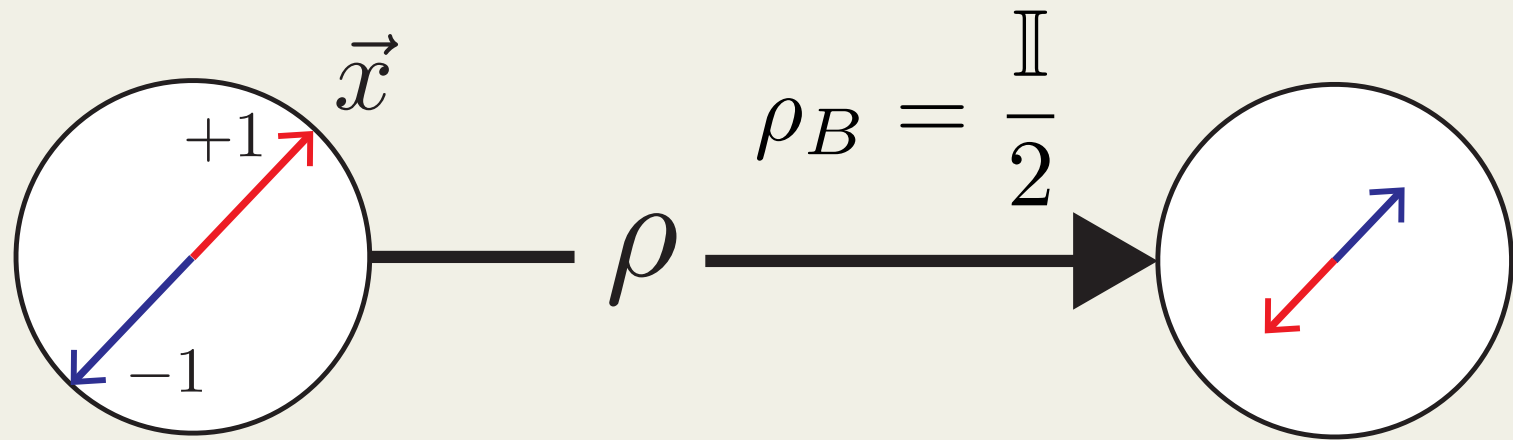
LHS model $\iff \rho$ unsteerable
 for all \vec{x} $\implies \rho$ Bell local

Example: Werner state



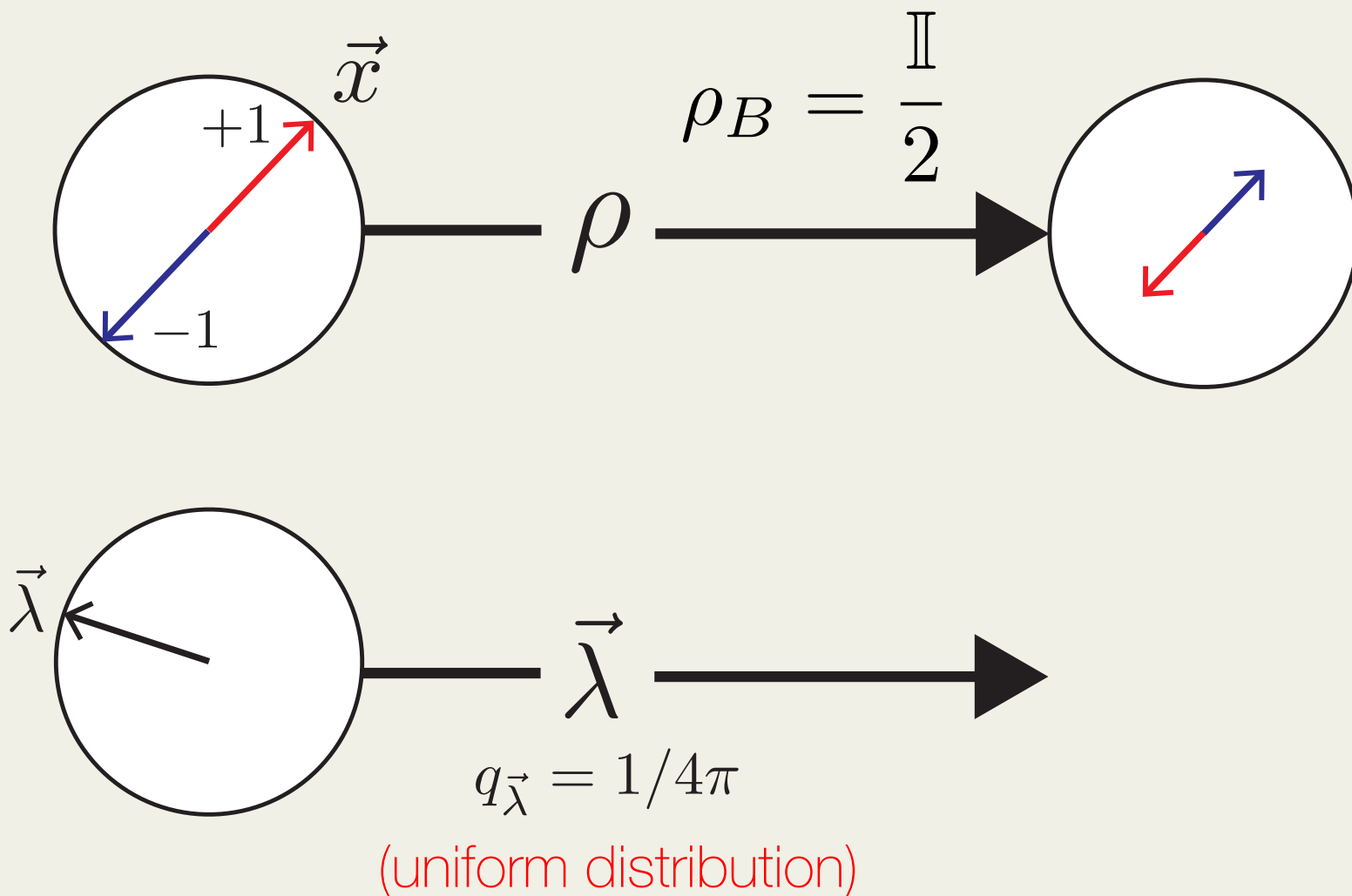
$$\rho = \frac{1}{2} |\psi^-\rangle \langle \psi^-| + \frac{1}{2} \frac{\mathbb{I}}{4}$$

Example: Werner state

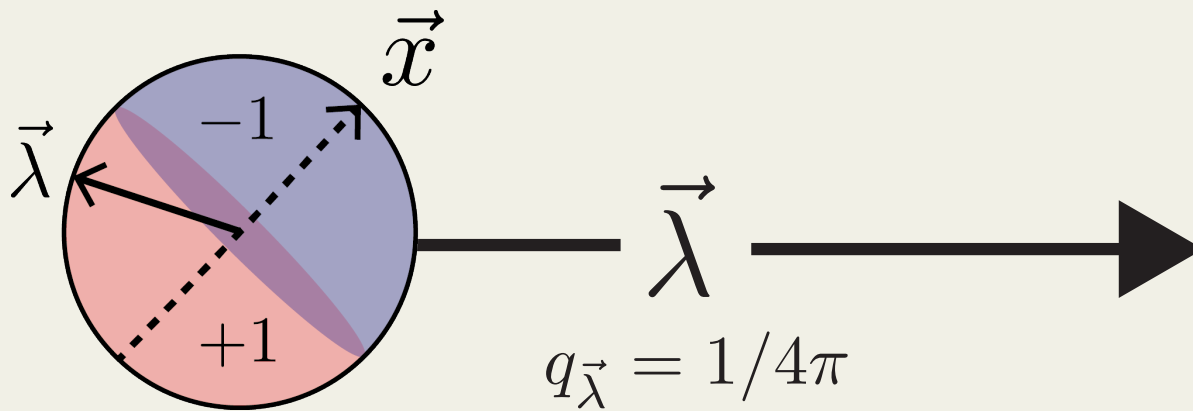
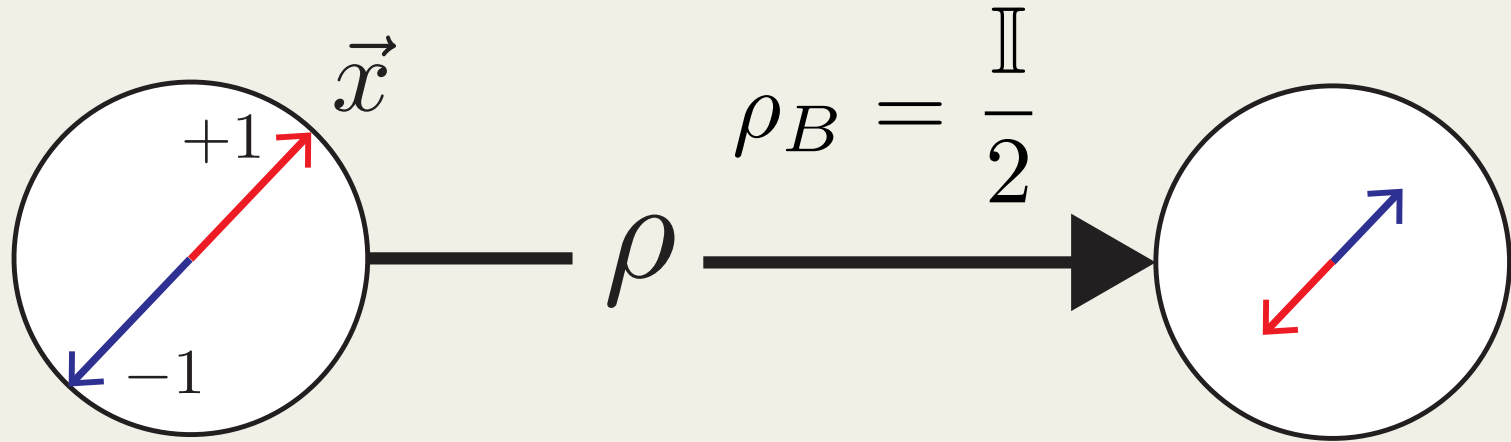


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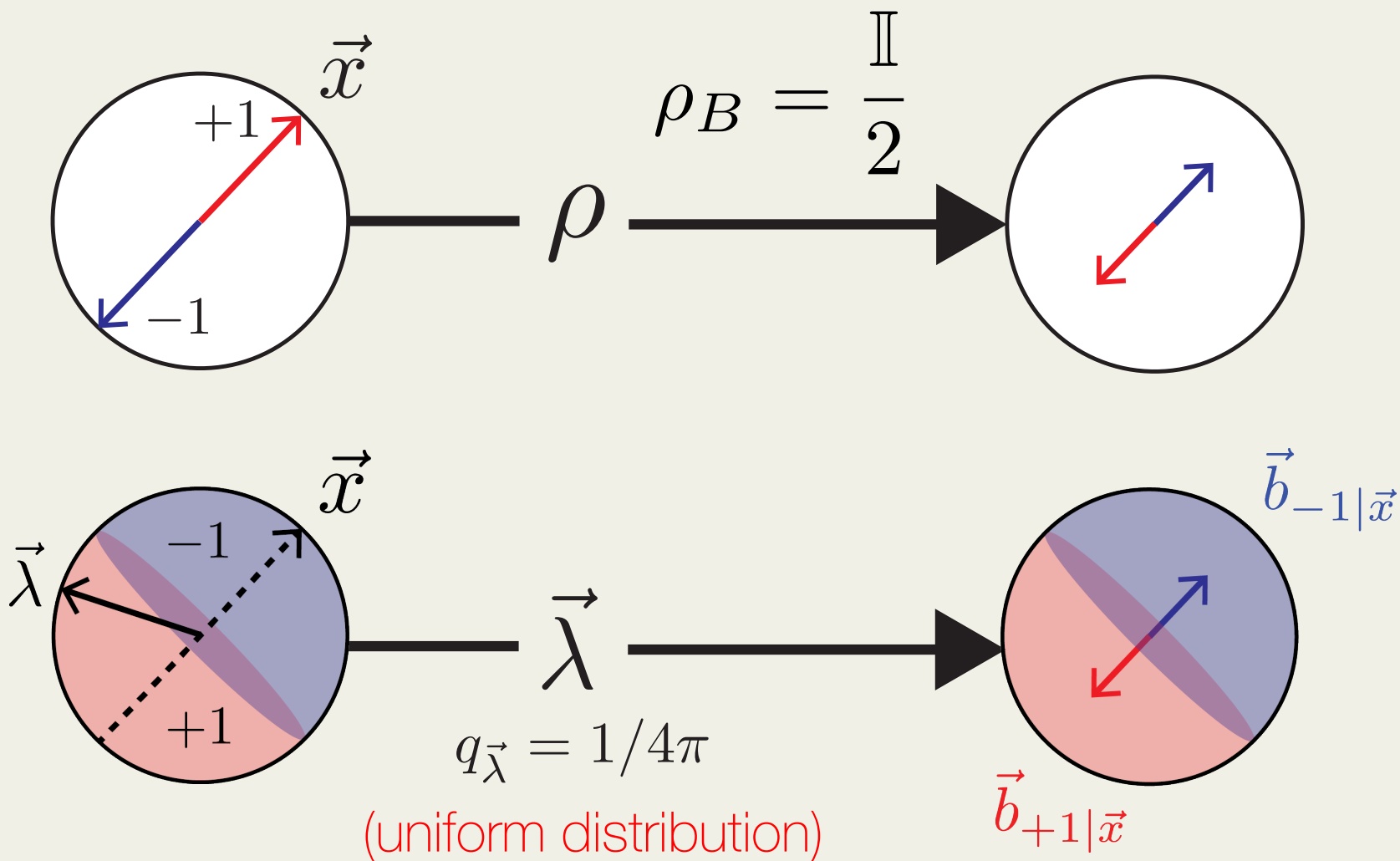


Example: Werner state



(uniform distribution)

Example: Werner state



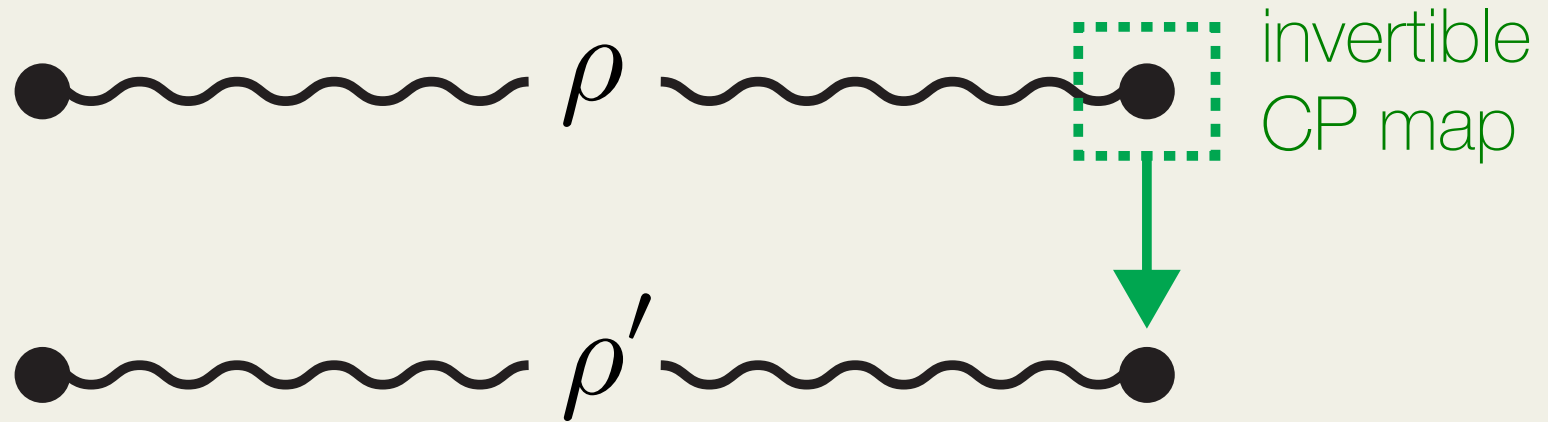
How can we construct LHS models for states with less symmetry?

Equivalent states



Quintino et al. PRA **92**, 032107 (2015)
Gallego, Aolita, PRX **5**, 041008 (2015)

Equivalent states



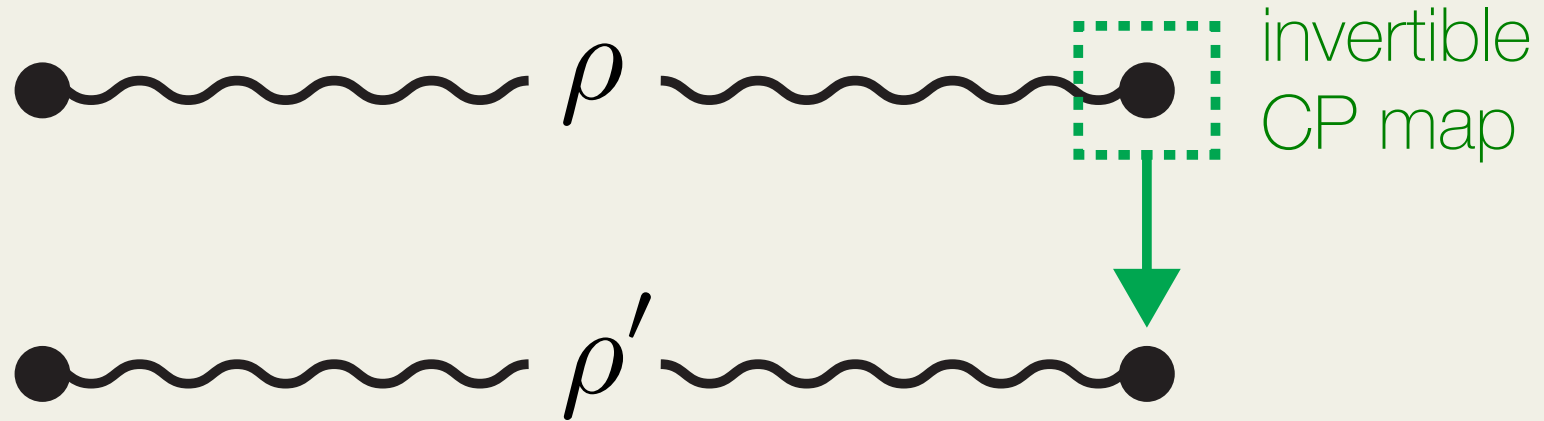
$$\rho \text{ unsteerable} \iff \rho' \text{ unsteerable}$$

Equivalent states



$$\rho' = \frac{\mathbb{I} \otimes \rho_B^{-\frac{1}{2}} \rho \mathbb{I} \otimes \rho_B^{-\frac{1}{2}}}{\text{Tr} \left[\mathbb{I} \otimes \rho_B^{-\frac{1}{2}} \rho \mathbb{I} \otimes \rho_B^{-\frac{1}{2}} \right]} \implies \rho'_B = \frac{\mathbb{I}}{2}$$

Equivalent states

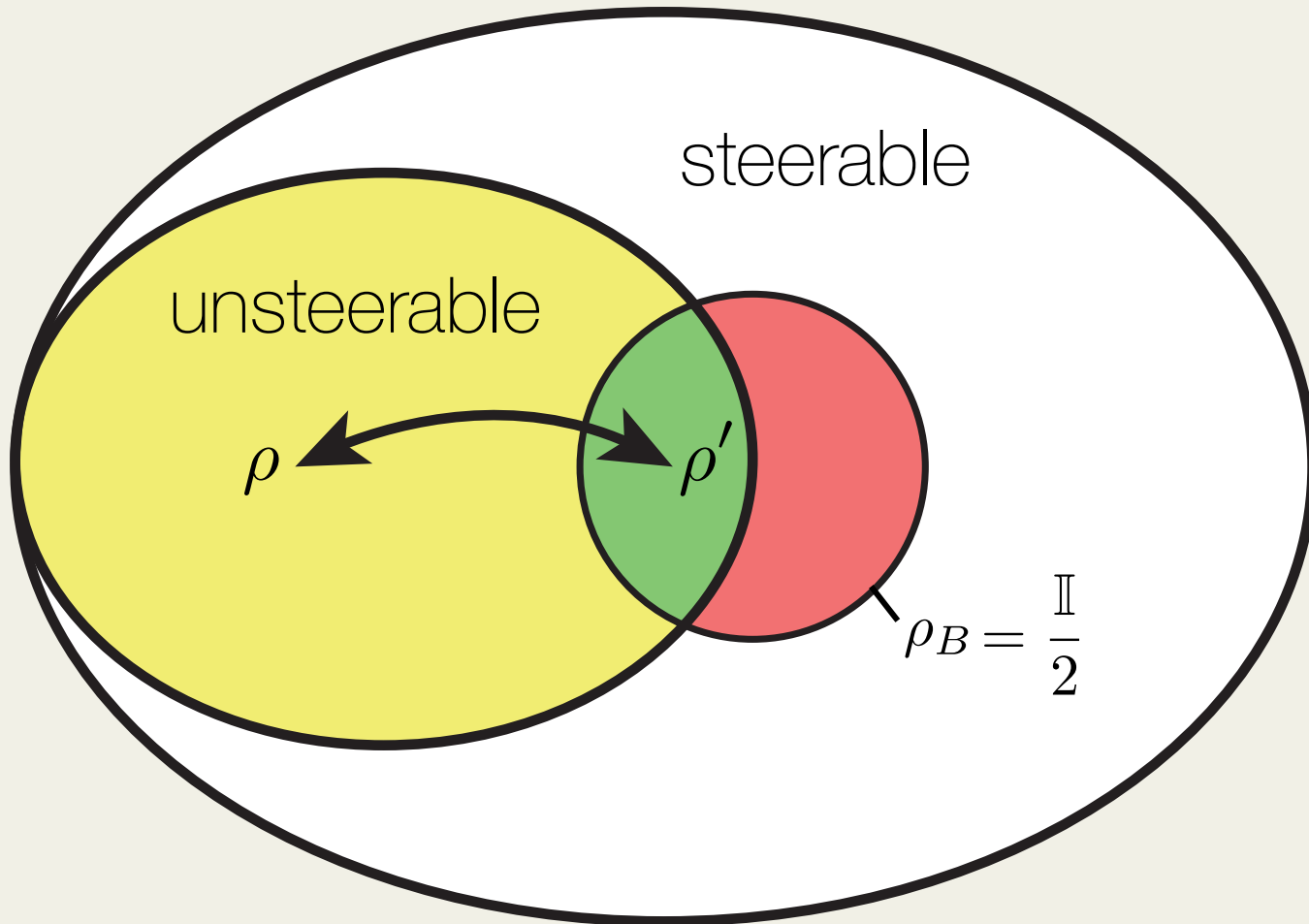


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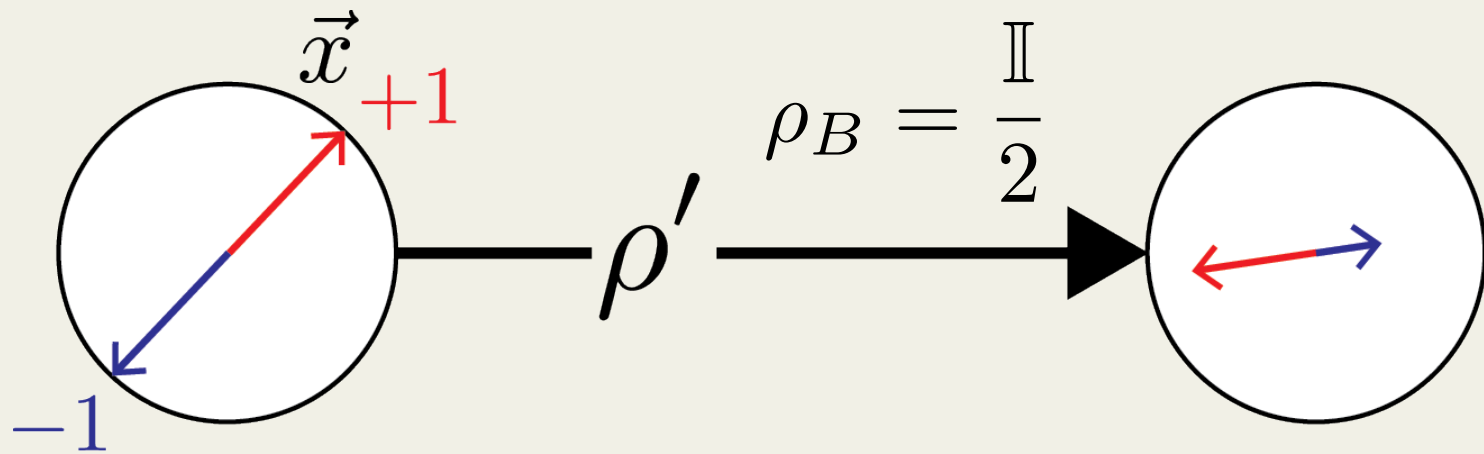
canonical
state

Quintino et al. PRA 92, 032107 (2015)
Gallego, Aolita, PRX 5, 041008 (2015)

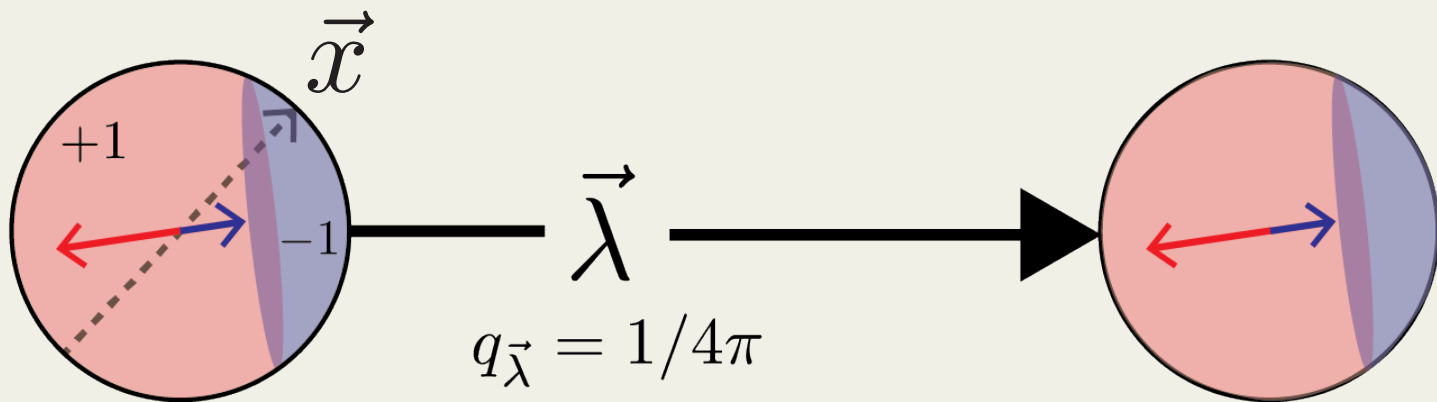
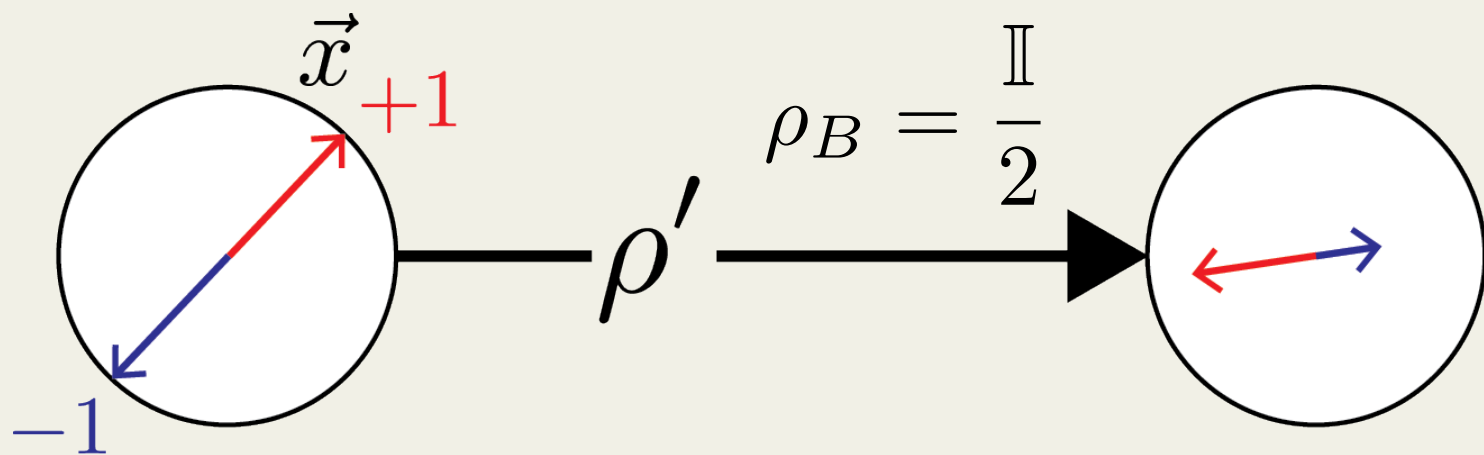
Canonical states



General LHS Model



General LHS Model

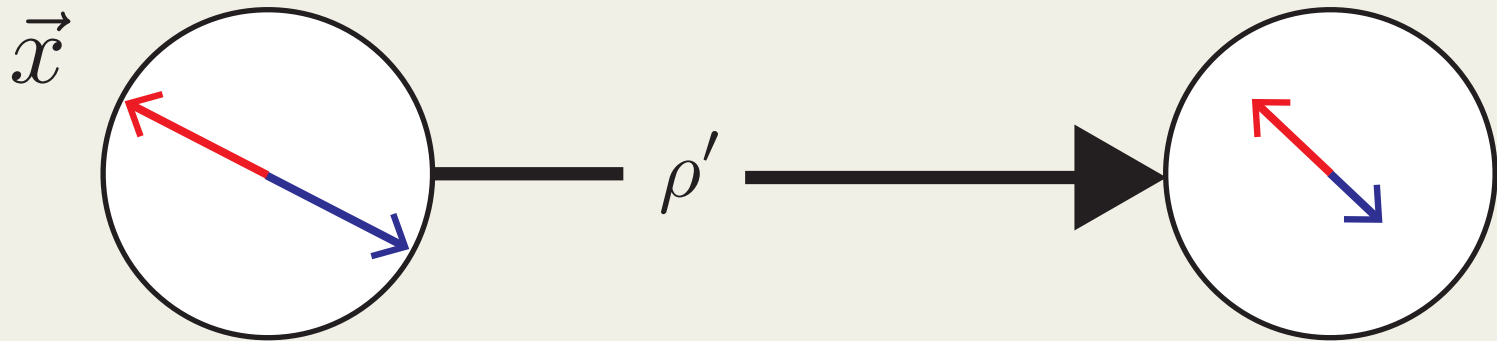


Criterion for unsteerability

$$\rho \rightarrow \rho' = \frac{1}{4} \left[\mathbb{I} + \vec{a} \cdot \vec{\sigma} \otimes \mathbb{I} + \sum_i T_i \sigma_i \otimes \sigma_i \right]$$

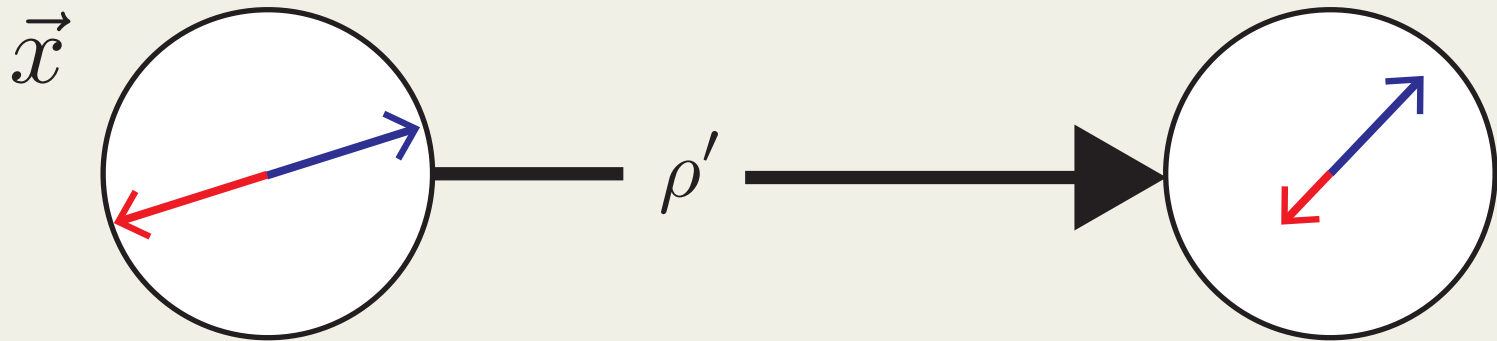
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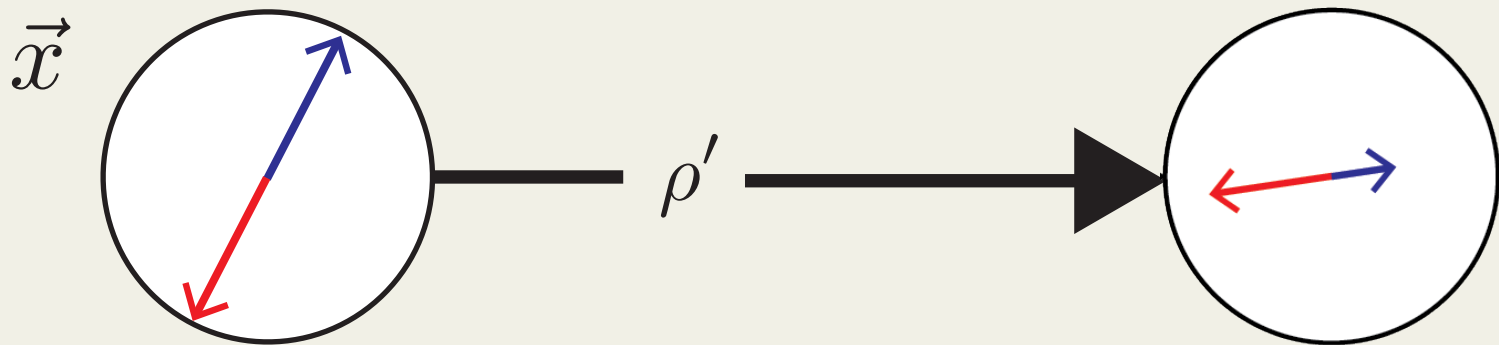
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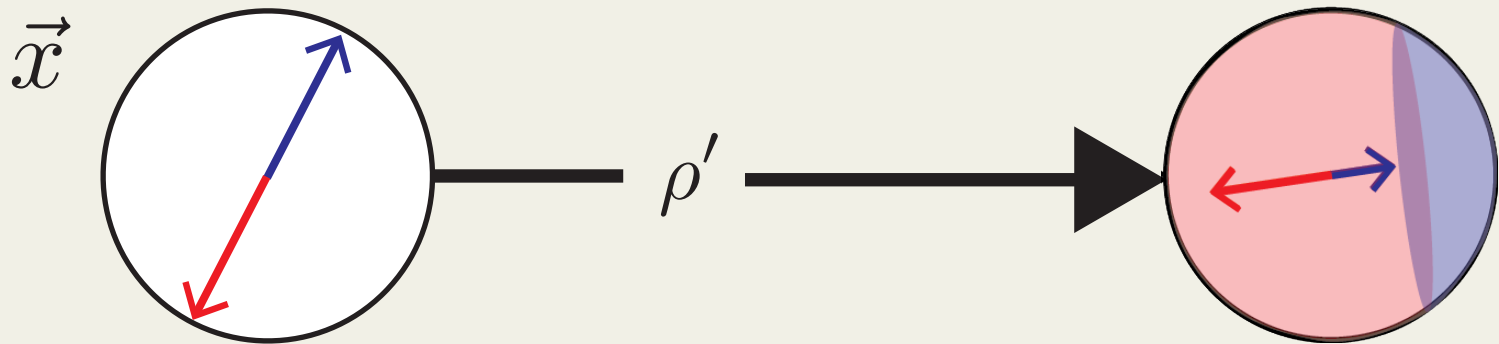
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Criterion for unsteerability

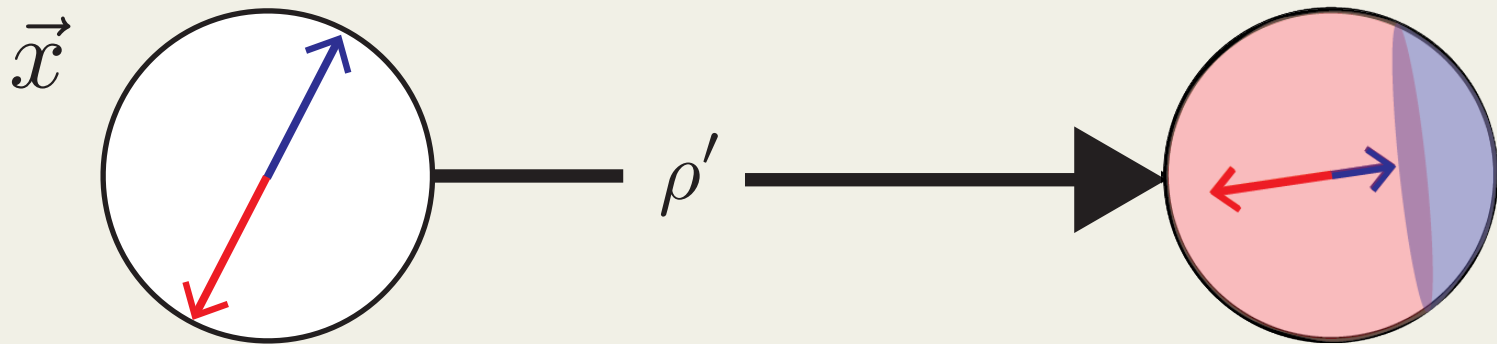
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$$(\vec{a} \cdot \vec{x})^2 + 2||T\vec{x}|| \leq 1$$

Criterion for unsteerability

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ρ unsteerable if

$$\max_{\vec{x}} \left[(\vec{a} \cdot \vec{x})^2 + 2 ||T\vec{x}|| \right] \leq 1$$

Example

$$\rho = \alpha |\psi_\theta\rangle\langle\psi_\theta| + (1 - \alpha)\rho_A \otimes \frac{\mathbb{I}}{2}$$

$$|\psi_\theta\rangle = \cos\theta|01\rangle - \sin\theta|10\rangle$$

$$\rho_A = \text{Tr}_B |\psi_\theta\rangle\langle\psi_\theta|$$

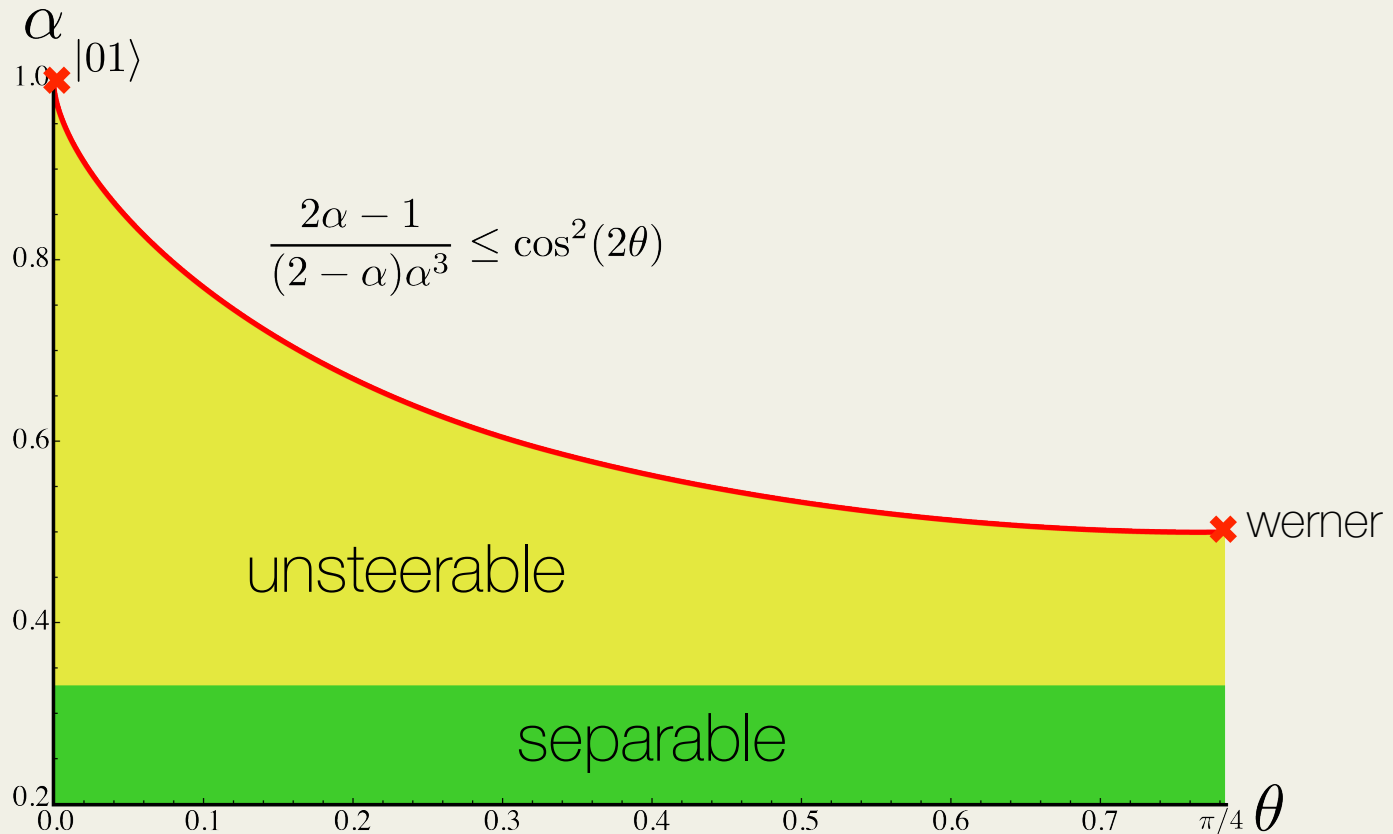
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Applications

Sunday 9am

[arXiv:1510.06722](#) [pdf, other]

Incompatible quantum measurements admitting a local hidden variable model

[Marco Túlio Quintino](#), [Joseph Bowles](#), [Flavien Hirsch](#), [Nicolas Brunner](#)

Applications

Sunday 9am

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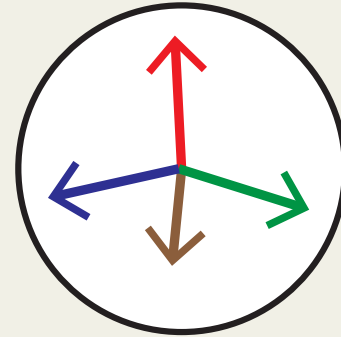
[arXiv:1511.08401](#) [pdf, other]

Genuinely multipartite entangled quantum states with fully local hidden variable models and hidden multipartite nonlocality

[Joseph Bowles](#), [Jérémie Francfort](#), [Mathieu Fillettaz](#), [Flavien Hirsch](#), [Nicolas Brunner](#)

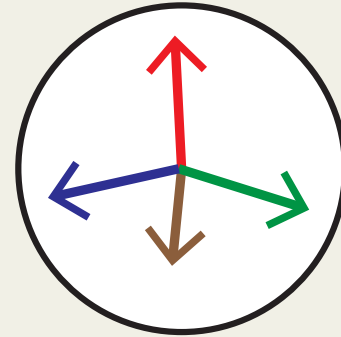
Open questions

- POVMs?



Open questions

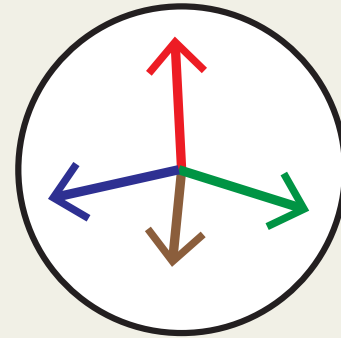
- POVMs?



- necessary + sufficient?

Open questions

- POVMs?



- necessary + sufficient?

[arXiv:1510.06721](#) [pdf, other]

Sufficient criterion for guaranteeing that a two-qubit state is unsteerable

Joseph Bowles, Flavien Hirsch, Marco Túlio Quintino, Nicolas Brunner