# Test of Genuine Multipartite Nonlocality for quantum composite systems

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Abstract:

All pure entangled states exhibit standard nonlocality. It is an open problem whether all pure genuine(full) multipartite entangled states are genuine nonlocal. We propose a set of conditions on the joint probabilities as a test of genuine multipartite nonlocality without inequality. A pass of our test by a state therefore indicates that this state cannot be simulated by any nonsignaling local models i.e. the state exhibits genuine multipartite nonlocality. It turns out that all entangled symmetric n-qubit ($n \geqslant 3$) states pass our test and therefore are n-way nonlocal. The talk is based on the papers: Phys. Rev. Lett. **109**, 120402 (2012); Phys. Rev. Lett. **112**, 140404 (2014).

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# Main focus

What is nonlocality?
Kinematic

Macro-nonlocality and micro-nonlocality

Einstein locality and Bell nonlocality

Entangled states and **multipartite nonlocality**

**Test of nonlocality for multipartite systems.**

# Background

What is **nonlocality** in physics?

Or what is locality in physics?

All fundamental forces are **local** interaction, **dynamic**.

States of a physical system need not be **local**, **kinematic.**

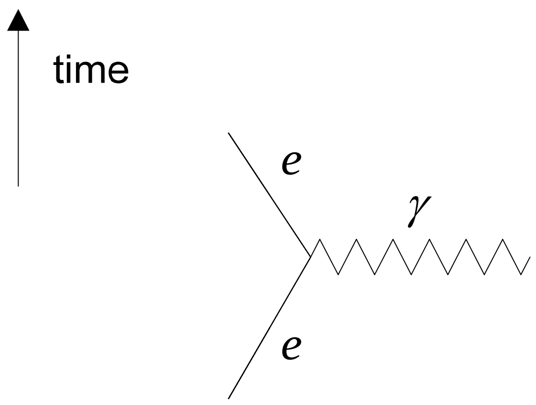
**Quantum mechanics is nonlocal, nonlocal here refers to state arising from to linear superpostion of oher states.**

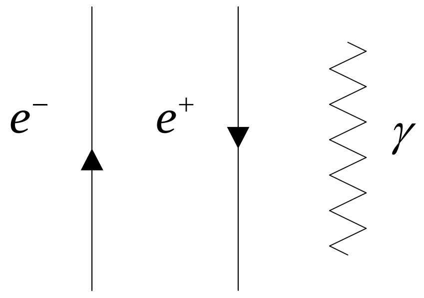
# Background

All fundamental forces are local interaction, dynamic.

Quantum electrodynamics (QED)

All *electromagnetic* phenomena are ultimately reducible to following elementary process (primitive vertex)




$$L = \bar{\psi}\gamma^{\mu} D_{\mu}\psi - \frac{1}{4} F_{\mu\nu}F^{\mu\nu} + m\bar{\psi}\psi$$

$$= \bar{\psi}\gamma^{\mu}\partial_{\mu}\psi - ie\bar{\psi}\gamma^{\mu}\psi A_{\mu} - \frac{1}{4} F_{\mu\nu}F^{\mu\nu} + m\bar{\psi}\psi$$

Interaction vertex $\bar{\psi}\gamma^{\mu}\psi A_{\mu} = j^{\mu} A_{\mu}$

and $F_{\mu\nu} = \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu}$

# Background

All fundamental forces are local, interaction terms local , dynamic
Typical symmetry group SU(N), N=3 for quantum chromodynamics

Global transformation

$$U(\theta) = \exp\left(-\frac{i}{\hbar}\theta^a T_a\right),$$ $T_a$ is group generator and $\theta^a$ rigid parameter.

**Local** transformation

$$U(\theta) = \exp\left(-\frac{i}{\hbar}\theta^a(x) T_a\right),$$ $T_a$ is group generator and $\theta^a(x)$ **localized** parameter.

*Now if we adopt the view that this arbitrary convention should be* ***independently chosen at every spacetime point,*** *then we are* ***naturally led to the concept of***

C N Yang

# Background

All fundamental interactions are local, and the force fields are gauge fields, Yang-Mills fields.

Once the symmetry group is **localized** , that is, gauged, the emergence of gauge field is natural.

Gauged symmetry dictates intecation,

C N Yang

# Background

Nonlocal interaction in quantum mechanics, examples

Aharonov effect (topological)

Berry phase (geometric)

# Background

Locality is a basic assumption in quantum field theory:

The commutator or anticommutator of two local field operators vanishes for space-like separation,

$$\left[\phi(x), \quad \pi(x')\right] = 0, \qquad \text{for} \qquad (x - x')^2 < 0$$

This requirement, also known as microscopic causality principle, is the mathematical statement of the fact that no signal can be exchanged between two points separated by a spacelike interval, and
**therefore measurements at such points cannot interfere**.

→ Einstein’s locality

# Einstein’s locality

After the famous EPR paper Phys Rev 1935

Einstein stated the principle of locality

“***The real factual situation of the system*** $S_2$ ***is independent of what is done with the system*** $S_1$ ***,***

***which is spatially separated from the former***”

# Bell's nonlocality

The Bell theorem and henceforth the Bell inequality

is a  test for nonlocality and is formulated for

**bipartite**  physical system.

The **state exhibits nonlocality** if the correlations between measurement settings and measurement results **violate** a Bell inequality.

Question: Can this be generalized to **multipartite**

physical systems?

# Entangled states

Entanglement means that the state of the quantum system cannot be written as a mixture of product states of its constituent subsystems.

Entanglement and nonlocality are the two different yet main concepts in the quantum information sciences.

Although it is immediately clear that **entanglement is necessary for nonlocality**, a detailed quantitative relation between these two concepts is not yet well established.

# Entanglement versus Nonlocality

- To exhibit Bell's nonlocality, the state has to be entangled since there is no violation for separable states.
- However, there exist entangled (mixed) states that can be simulated by LHV models, i.e., do not violate any Bell inequality and thus do not exhibit Bell's nonlocality. [Werner PRA89]

# Gisin's theorem: entangled states and the Bell nonlocality

- In 1991, Gisin' theorem[1] is presented:

Any pure entangled state of two particles violates a Bell inequality for two-particle correlation functions.

That is, every pure bipartite entangled state in two dimensions violates the CHSH inequality.

[1] N. Gisin, Phys Lett A **154**, 201 (1991).

Question

Can the Gisin theorem be generalized to three-qubit

pure entangled states such as the GHZ states?

**Greenberger-Horne-Zeilinger (GHZ) paradox**

# Known Generalized Bell inequalities

- Bell inequalities for N qubits

**MABK inequalities (1990's)**

N. D. Mermin, Phys. Rev. Lett. **65**, 1838 (1990); M. Ardehali, Phys. Rev. A **46**, 5375 (1992); A. V. Belinskii and D. N. Klyshko, Phys. Usp. **36**, 653 (1993).

**Zukowski-Brukner inequalities (2002)**

M. Zukowski and C. Brukner, Phys. Rev. Lett. **88**, 210401 (2002).

Remark: all the Bell inequalities were derived in terms of N-site correlation functions.

- Bell inequalities for 2 quNits

**CGLMP inequalities** [equivalent correlation-function form was given in PRL, 92, 130404 (2004)]

D. Collins, N. Gisin, N. Linden, S. Massar, and S. Popescu, Phys. Rev. Lett. **88**, 040404 (2002).

A. Acin, J L Chen, N Gisin, D Kaszlikowski, L C Kwek, C H Oh and Zukowski Marek, PRL, 92, 250404 (2004).

# Examples of Bell's inequalities

In general Bell's inequality is characterized by 3 parameters (*n,m,d*) : each of *n* observers measures *m* observables with *d* outcomes. We consider only (n,2,2) scenario here.

✧ lauser- orne- himony- olt (CHSH) inequality is complete* for (2,2,2) scenario [CHSH PRL69].

✧ ermin- rdehali- elinskii- lyshko (MABK) inequality (n,2,2) is a generalization of Bell-CHSH inequality to multipartite systems. [ M PRL90, A PRA92, BK PU93]

✧ erner- olf- ukowski- rukner (WWZB) inequality is a complete set of Bell's inequalities for *n*-qubit correlations for (n,2,2) scenario [WW PRA2001, ZB PRL2002]

* If a Bell inequality is a necessary and sufficient condition for a local realistic description of the correlation functions, we call it is complete.

# MABK inequalities (1990's)
# Zukowski-Brukner inequalities(2002)

Gisin and Scarani noticed that there exist pure entangled states of 3-qubits that do not violate any of the inequality of this family.

The result came as a surprise since it was believed that all 3 qubit pure states should violate the general correlation-Bell inequality.

# Gisin's Theorem for 3 Qubits, Chen JL(2004)

All generalized GHZ state (*) for three-qubit systems violate Bell inequality

$$\left|\psi\right\rangle_{GHZ} = \cos\xi\left|000\right\rangle + \sin\xi\left|111\right\rangle \quad (*)$$

A single Bell's inequality is violated by all entangled pure states of three qubits.

Jingling Chen, Chunfeng Wu, L.C. Kwek and C.H. Oh, Phys. Rev. Lett. , **93**, 140407, (2004)

# Recent results

A new Hardy-type test to detect genuine multipartite nonlocality.

All pure entangled symmetric n-qubit (n >2) states are genuine multipartite nonlocal.

Chen-Yu-Zhang-Lai-Oh
Phys. Rev. Lett. **112**, 140404 (2014).

# Entangled states(definition)

- For n-partite pure state $|\psi\rangle$

If $|\psi\rangle \neq |\varphi_1\rangle \otimes |\varphi_2\rangle \otimes \ldots \otimes |\varphi_n\rangle$, $|\varphi_k\rangle$ is a pure state of k-th partite system then $|\psi\rangle$ is an **entangled state**.

- For n-partite mixed state $\rho$

A mixed state of n systems is **entangled** if it cannot be written as a convex combination of product state, i.e,

$$\rho \neq \sum_i p_i \, \rho_i^1 \otimes \rho_i^2 \otimes \ldots \otimes \rho_i^n,$$

- $\rho_i^k$ is a pure (or mixed) state of k-th partite system
- $\sum_i p_i = 1$

# Genuinely multipartite entangled states(definition)

- An n-partite pure state $|\psi\rangle$ is called genuinely multipartite entangled if $|\psi\rangle \neq |\varphi_\alpha\rangle \otimes |\varphi_{\overline{\alpha}}\rangle, \forall \alpha \neq \phi, \alpha \subset I$
  - $I = \{1,2,\ldots,n\}$
  - $\alpha, \overline{\alpha} = I \setminus \alpha :$ nonempty proper subset of $I$
  - $|\varphi_\alpha\rangle, |\varphi_{\overline{\alpha}}\rangle$ are pure states of subsystem $\alpha, \overline{\alpha}$

- An n-partite mixed state $\rho$ is called genuinely multipartite entangled if

$$\rho \neq \sum_{\alpha \neq \phi, \alpha \subset I} \sum_i p_{\alpha,i}\, \rho_i^{\alpha} \otimes \rho_i^{\overline{\alpha}}$$

  - $\rho_i^{\alpha}, \rho_i^{\overline{\alpha}}$ are pure (or mixed) states on subsystem $\alpha, \overline{\alpha}$
  - $\sum_{\alpha \neq \phi, \alpha \subset I} \sum_i p_{\alpha,i} = 1$

- A state which is entangled but not genuinely multipartite entangled is called **partially entangled**.

# Local models for n-partite system

- Standard local model: joint probability $P(r_I \mid M_I)$ assumes

$$P(r_I \mid M_I) = \int \rho_\lambda \prod_{k=1}^{n} P_k(r_k \mid M_k, \lambda) d\lambda$$

  - $I = \{1,2,\ldots,n\}$
  - $M_k$ : measurement setting, $r_k$ : outcome, $k \in I$
  - $r_I = \{r_1,\ldots,r_n\}, M_I = \{M_1,\ldots,M_n\}$
  - $P_k(r_k \mid M_k, \lambda)$ is the probability of observer $k$ measuring

observable $M_k$ with outcome $r_k$ for a given hidden variable $\lambda$ distributed according to $\rho_\lambda$ with normalization $\int \rho_\lambda d\lambda = 1$

# Standard local realistic models

Each observer cannot have nonlocal correlations, with any other distant observers, the **joint probability distribution can be fully factorized** and written as, e g, for 3-particle system,

$$p(abc|\, xyz) = \int P_\lambda(a\,|\,x) P_\lambda(b\,|\,y) P_\lambda(c\,|\,z) \rho_\lambda d\lambda.$$

- *xyz: measurement settings (input)*
- *abc: the outcomes of the measurements (output)*

# Local models for n-partite system

- The most general **hybrid local-nonlocal** model: joint probability distribution can be written as

$$P(r_I \mid M_I) = \sum_{\alpha\neq\phi,\alpha\subset I}\int \rho_{\alpha,\lambda} P_\alpha(r_\alpha \mid M_\alpha,\lambda) P_{\overline{\alpha}}(r_{\overline{\alpha}} \mid M_{\overline{\alpha}},\lambda)\,d\lambda$$

- $\alpha$ : a nonempty proper subset of $I$
- $\overline{\alpha} = I \setminus \alpha$ : also a nonempty proper subset of $I$
- $r_\alpha, r_{\overline{\alpha}}$ : outcomes, $M_\alpha, M_{\overline{\alpha}}$ : observables
- $P_\alpha(r_\alpha \mid M_\alpha,\lambda)$ is the joint probability of observer $k$ measuring observable $M_k$ with outcome $r_k$ for all observer $k \in \alpha$ for a given hidden variable $\lambda$ distributed according to $\rho_{\alpha,\lambda}$

# The definition of standard nonlocality

For "standard" nonlocal correlations, the joint probability distribution **cannot** be written as

$$p(abc \mid xyz) = \int P_\lambda(a \mid x) P_\lambda(b \mid y) P_\lambda(c \mid z) \rho_\lambda d\lambda.$$

however as **partial factorization is not excluded**, e.g.,

$$p(abc \mid xyz) = \int P_\lambda(ab \mid xy) P_\lambda(c \mid z) \rho_\lambda d\lambda.$$

- *xyz: measurement settings (input)*
- *abc: the outcomes of the measurements (output)*

# Genuine multipartite nonlocality

— **“standard” nonlocal** correlations, the joint probability distribution **cannot** be written as

$$p(abc \mid xyz) = \int P_\lambda(a \mid x) P_\lambda(b \mid y) P_\lambda(c \mid z) \rho_\lambda d\lambda$$

— **genuine multipartite nonlocal** correlations, **cannot** be written as [Svetlichny, PRD87]

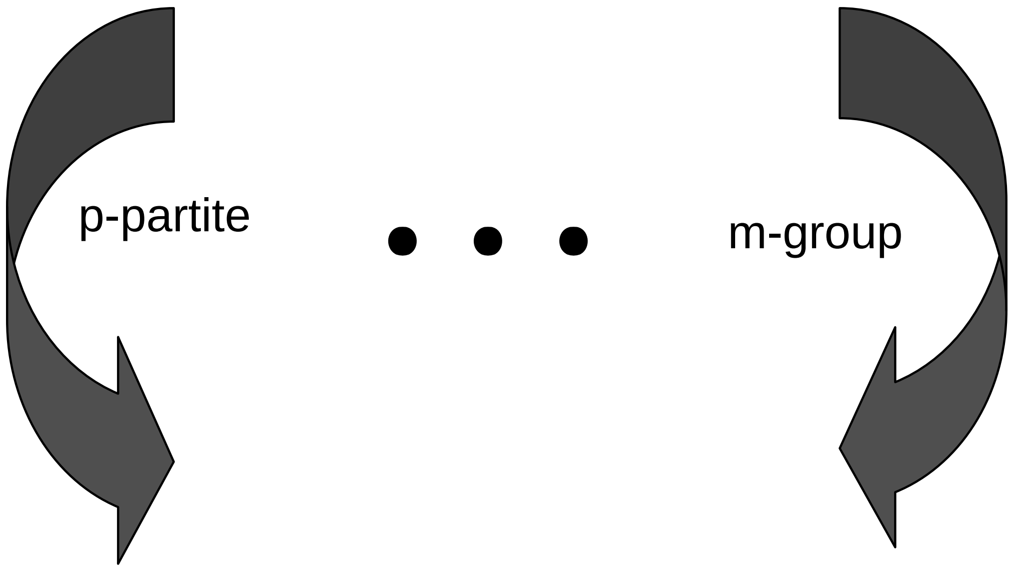
$$p(abc \mid xyz) = \int P_\lambda(a \mid x) P_\lambda(bc \mid yz) \rho_\lambda d\lambda +$$

$$\int P_u(b \mid y) P_u(ac \mid xz) \rho_u du + \int P_v(c \mid z) P_v(ab \mid xy) \rho_v dv$$

- *xyz: measurement settings (input)*
- *abc: the outcomes of the measurements (output)*

# Different classes of nonlocality and entanglement

- standard nonlocality (entanglement) -- weakest




- genuine multipartite nonlocality (entanglement) -- strongest

# Svetlichny inequalities

- Define recursively the Svetlichny polynomials as

$$S_n = \frac{1}{2}(S_{n-1}A'_n + S'_{n-1}A_n) \text{ for } n \geq 3,$$

$$S_2 = (A_1A_2 + A_1A'_2 + A'_1A_2 - A'_1A'_2)/2 = CHSH,$$

$$S'_2 = (A_1'A'_2 + A'_1A_2 + A_1A'_2 - A_1A_2)/2$$

$$Examples: S_3 = (-A_1A_2A_3 + A_1A_2A'_3 + A_1A'_2A_3 + A'_1A_2A_3 + A_1A_2'A'_3 + A_1'A'_2A_3 + A'_1A_2A'_3 - A'_2A'_2A'_3)/4$$

- Svetlichny inequality $\langle S_n \rangle \leq 1$ holds for any bipartite probability distribution.
- For GHZ state: $\langle S_n \rangle_{GHZ} = \sqrt{2}$ by proper choosing measurement settings.
- There only exist limited states that violate Svetlichny inequalities. [S PRD 87, CGPRS PRL02, SS PRL02.]

# New test

$$P(0_I \mid a_I) > 0 \qquad \text{(1a)}$$

$$P(0_I \mid b_k a_{\bar{k}}) = 0, \forall k \in I \qquad \text{(1b)}$$

$$P(1_{k'} 1_k 0_{I \setminus \{k',k\}} \mid b_{k'} b_k a_{I \setminus \{k',k\}}) = 0, \forall k \in I \setminus \{k'\} \qquad \text{(1c)}$$

- Proposition. Any probability distribution that satisfies Eq. (1) is genuine multipartite nonlocal.

[Chen-Yu-Zhang-Lai-Oh, Phys. Rev. Lett. **112**, 140404 (2014) ]

# Outline of the proof

- ➢ via **reductio ad absurdum**

- The most general hybrid local-nonlocal probability distribution can be written as

$$P(r_I \mid M_I) = \sum_{\alpha \neq \phi, \alpha \subset I} \int \rho_{\alpha,\lambda} P_\alpha(r_\alpha \mid M_\alpha, \lambda) P_{\overline{\alpha}}(r_{\overline{\alpha}} \mid M_{\overline{\alpha}}, \lambda) d\lambda \quad \text{(p0)}$$

  - $\alpha$ : nonempty proper subset of $I$
  - $r_\alpha, r_{\overline{\alpha}}$ : outcomes, $M_\alpha, M_{\overline{\alpha}}$ : observables

- Suppose a probability distribution satisfies Eq.(1) BUT is not genuine multipartite nonlocal, i.e., it has the form of Eq. (p0).

# Outline of the proof

- From Eq. (1a) there must exist some $\alpha_0$ and $\lambda_0$

$$P_{\alpha_0}(0_{\alpha_0} \mid a_{\alpha_0}, \lambda_0) > 0 \text{ and } P_{\overline{\alpha}_0}(0_{\overline{\alpha}_0} \mid a_{\overline{\alpha}_0}, \lambda_0) > 0 \quad \text{(p1)}$$

- From Eq. (1b)

$$P_{\alpha_0}(0_k 0_{\alpha_0 \backslash k} \mid b_k a_{\alpha_0 \backslash k}, \lambda_0)\, P_{\overline{\alpha}_0}(0_{\overline{\alpha}_0} \mid a_{\overline{\alpha}_0}, \lambda_0) = 0, \forall k \in \alpha_0$$

$$P_{\overline{\alpha}_0}(0_k 0_{\overline{\alpha}_0 \backslash k} \mid b_k a_{\overline{\alpha}_0 \backslash k}, \lambda_0)\, P_{\alpha_0}(0_{\alpha_0} \mid a_{\alpha_0}, \lambda_0) = 0, \forall k \in \overline{\alpha}_0 \quad \text{(p2)}$$

- By combining (p1) and (p2) we get

$$P_{\alpha_0}(0_k 0_{\alpha_0 \backslash k} \mid b_k a_{\alpha_0 \backslash k}, \lambda_0) = 0, \forall k \in \alpha_0 \quad \text{(p3a)}$$

$$P_{\overline{\alpha}_0}(0_k 0_{\overline{\alpha}_0 \backslash k} \mid b_k a_{\overline{\alpha}_0 \backslash k}, \lambda_0) = 0, \forall k \in \overline{\alpha}_0 \quad \text{(p3b)}$$

# Outline of the proof

- Suppose $k' \in \overline{\alpha}_0,\ j \in \alpha_0$, from Eq. (1c) we get

$$P_{\alpha_0}(1_j 0_{\alpha_0\backslash j} \mid b_j a_{\alpha_0\backslash j}, \lambda_0)\, P_{\overline{\alpha}_0}(1_{k'} 0_{\overline{\alpha}_0\backslash k'} \mid b_{k'} a_{\overline{\alpha}_0\backslash k'}, \lambda_0) = 0$$

- If $P_{\alpha_0}(1_j 0_{\alpha_0\backslash j} \mid b_j a_{\alpha_0\backslash j}, \lambda_0) = 0$, by combining Eq.(p3a) we find

$$P_{\alpha_0\backslash j}(0_{\alpha_0\backslash j} \mid a_{\alpha_0\backslash j}, \lambda_0)$$
$$= P_{\alpha_0}(0_j 0_{\alpha_0\backslash j} \mid b_j a_{\alpha_0\backslash j}, \lambda_0) + P_{\alpha_0}(1_j 0_{\alpha_0\backslash j} \mid b_j a_{\alpha_0\backslash j}, \lambda_0) = 0$$

However, $P_{\alpha_0\backslash j}(0_{\alpha_0\backslash j} \mid a_{\alpha_0\backslash j}, \lambda_0) \geq P_{\alpha_0}(0_{\alpha_0} \mid a_{\alpha_0}, \lambda_0) > 0$

$\Rightarrow$ a contradiction!

- If $P_{\overline{\alpha}_0}(1_{k'} 0_{\overline{\alpha}_0\backslash k'} \mid b_{k'} a_{\overline{\alpha}_0\backslash k'}, \lambda_0) = 0$, by combining Eq. (p3b) a similar contradiction can also be made.

# Nonsignaling correlations

Nonsignaling correlations: correlations obey nonsignaling principle, i.e., it is impossible to do instantaneous communication.

$$\sum_{a_k,\cdots,a_n} p(a_1,\cdots,a_k,\cdots,a_n \mid x_1,\cdots,x_k,\cdots,x_n)$$

$$= \sum_{a_k,\cdots,a_n} p(a_1,\cdots,a_k,\cdots,a_n \mid x_1,\cdots,x'_k,\cdots,x'_n)$$

$$= p(a_1,\cdots,a_{k-1} \mid x_1,\cdots,x_{k-1})$$

# Permutation symmetric states

Any n-qubit pure symmetric state can be written as

$$\left|\psi\right\rangle = \sum_{j=0}^{n} h_j \left|S(n, j)\right\rangle, \quad \left|S(n, j)\right\rangle = \sum_{\alpha \subseteq I, |\alpha| = j} \left|0_{\overline{\alpha}} 1_{\alpha}\right\rangle$$

Example

$$n = 2: \left|\psi\right\rangle = h_0\left|00\right\rangle + h_1\left(\left|01\right\rangle + \left|10\right\rangle\right) + h_2\left|11\right\rangle$$

Proposition: All pure multipartite entangled permutation symmetric n-qubit (n $\geqslant$ 3) states are genuine multipartite nonlocal.

# Permutation symmetric states

Proposition: All pure multipartite entangled permutation symmetric n-qubit ($n \geqslant 3$) states are genuine multipartite nonlocal.

We prove this proposition by showing that all pure entangled symmetric states pass our test by choosing the measurement setting suitably.

# Genuine multipartite nonlocality

- Our numerical results show that our test Eq.(1) can be satisfied by all pure genuine multipartite entangled states of three and four qubit systems.
- Conjecture: Our test can be satisfied by all pure genuine multipartite entangled states.

# 谢谢 **Thank you**