FWF Der Wissenschaftsfonds.

siqs

APCWQIS 2014, Cheng Kung University, 12.12.2014

Hybrid quantum circuits: Superconducting circuits coupling to other systems

Ze-Liang Xiang

Collaborations:

- P. Rabl (Atominstitut, TU Wien)
- J. Q. You (Beijing Computational Science Research Center)
- X.-Y. Lü, S. Ashhab, F. Nori (RIKEN)

TU WIEN TECHNISCHE UNIVERSITÄT WIEN Vienna University of Technology

ATOMINSTITUT

What's hybrid quantum circuits?

"Macroscopic" system:
Superconducting qubits

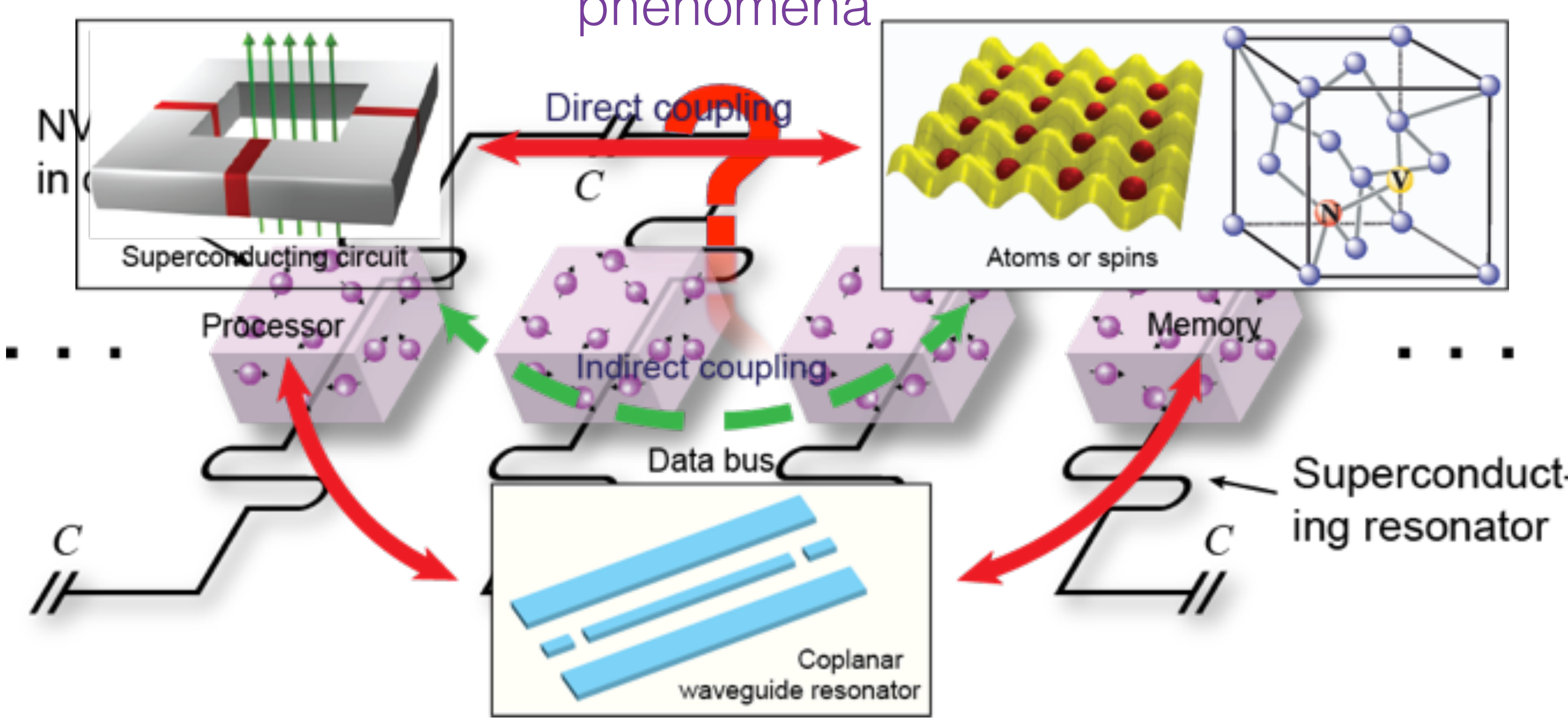
- Design flexibility
- Scalability
- Tunability
- Large coupling: fast
- Irreproducibility
- Short coherence time

"Microscopic" system:
Atoms, ions, spins

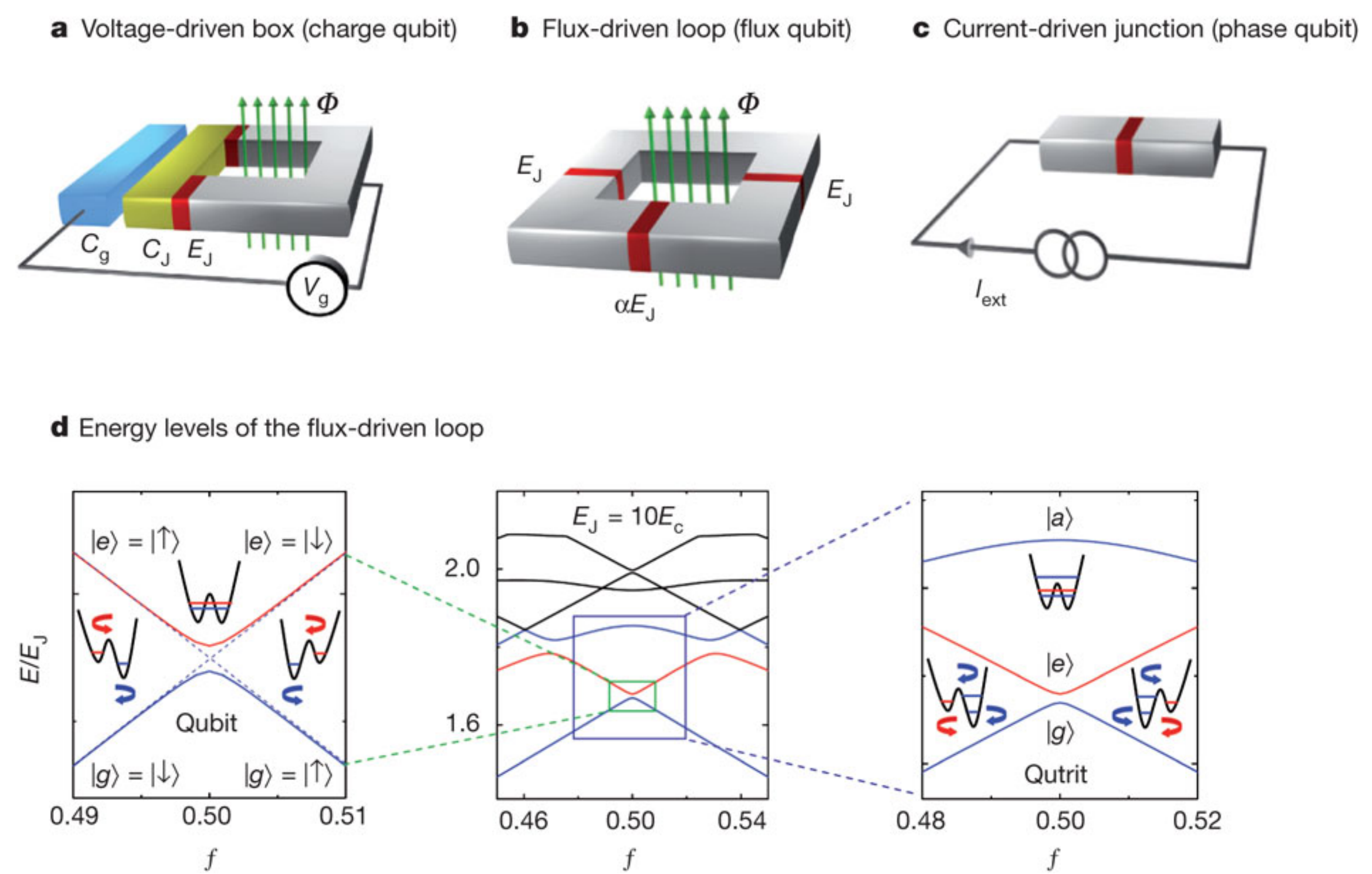
- Long coherence times
- Microwave to optical fields
- Reproducibility
- Nature given
- Small couplings: slow
- Limited scalability

Idea: Take only the best of both worlds

Q: Quantum simulation to new physical phenomena

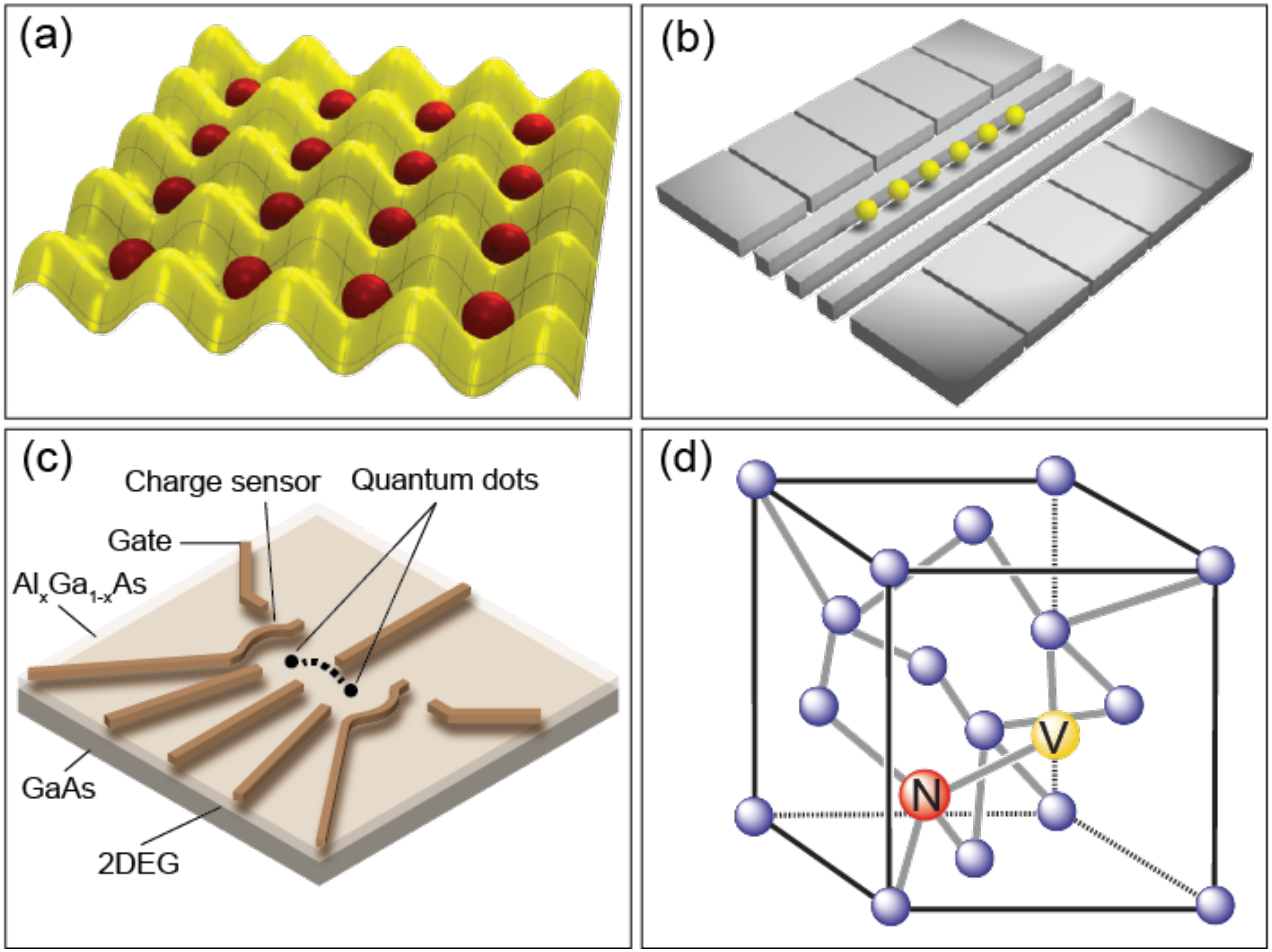


Elements: Superconducting qubits



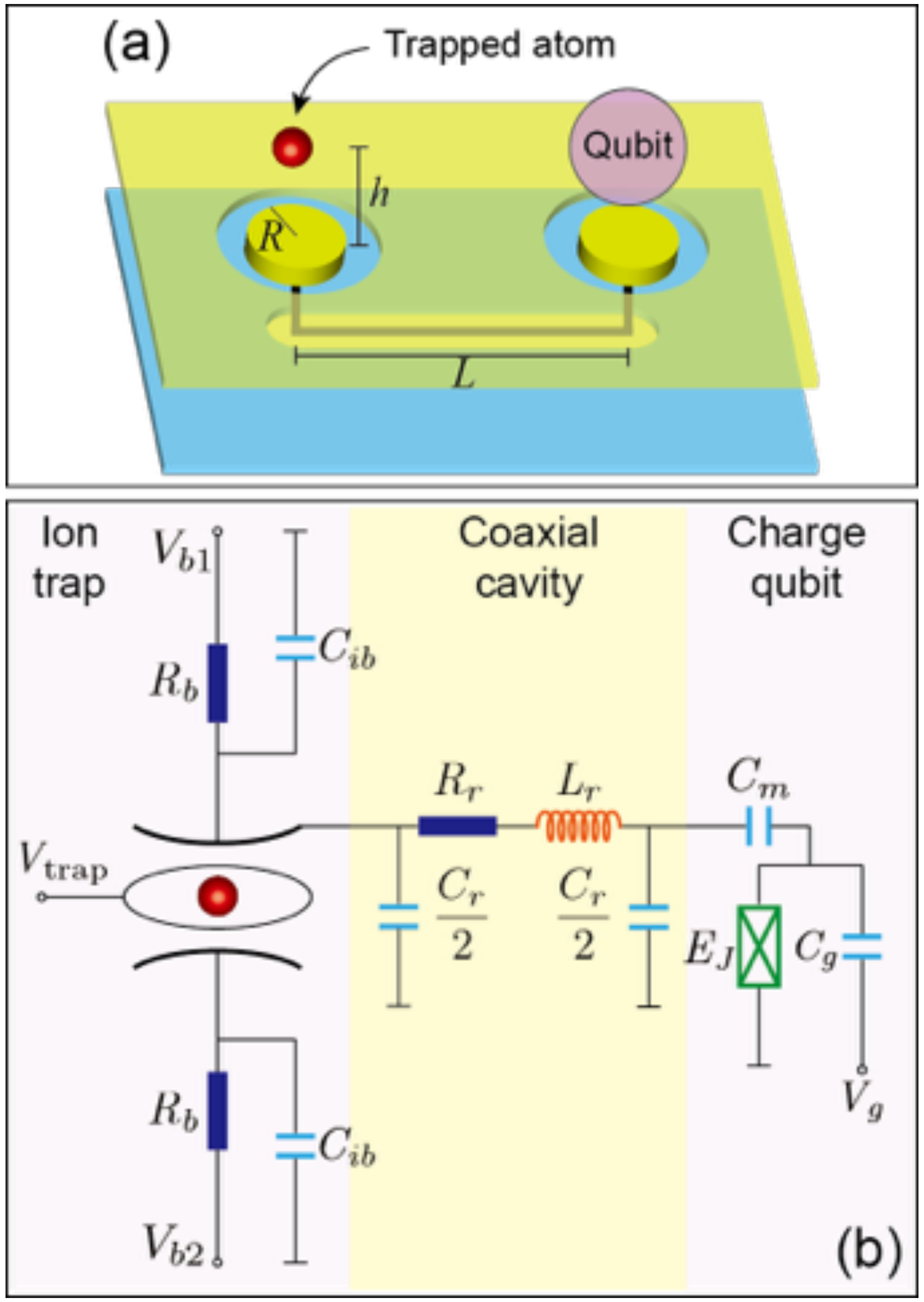
J. Q. You & F. Nori, Phys. Today (2005), Nature (2011)

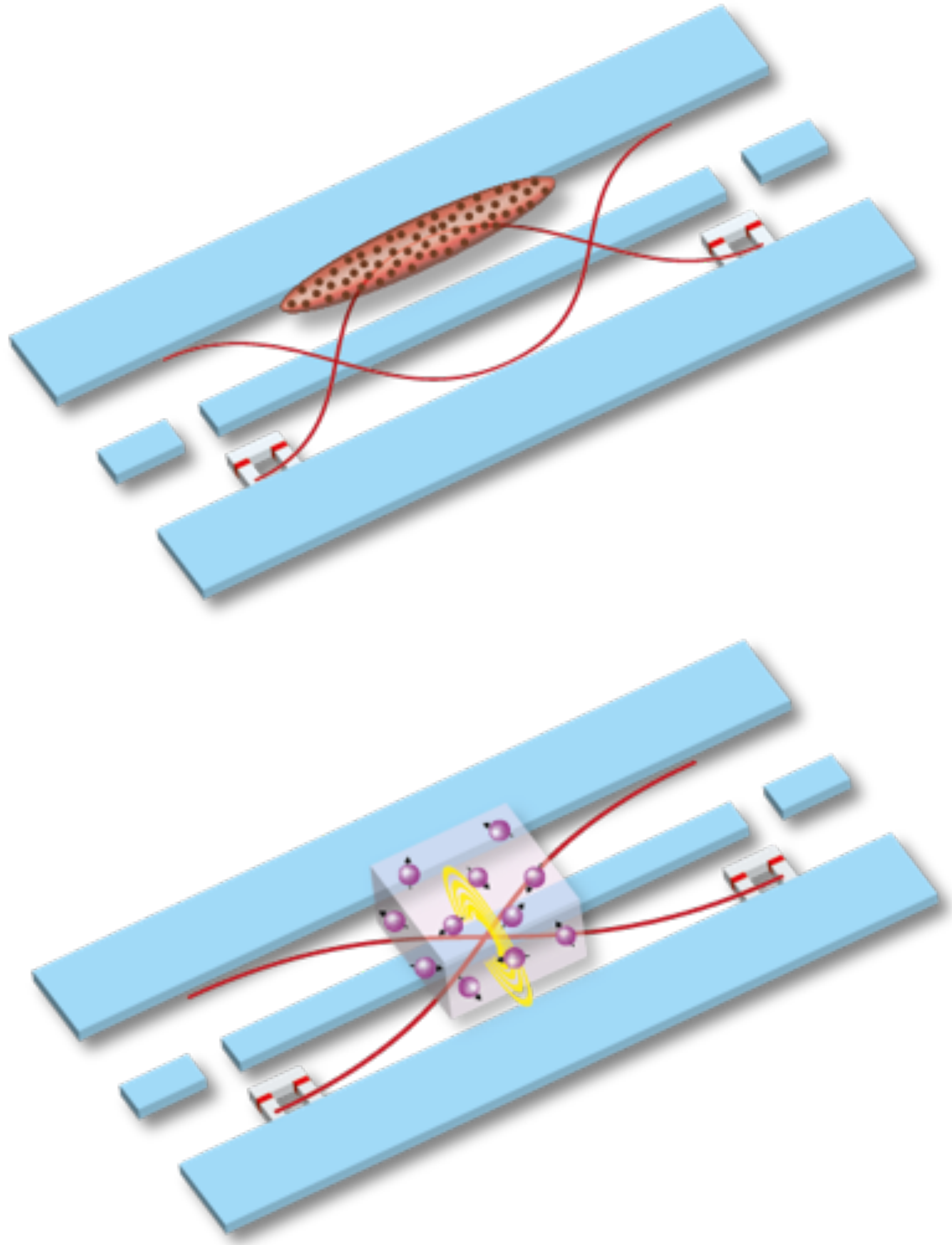
Elements: Atomic and spin qubits



I. Buluta et al., Rep. Prog. Phys. (2011); Z. -L. Xiang et al., Rev. Mod. Phys. (2013)

Hybrid quantum circuits: Indirect

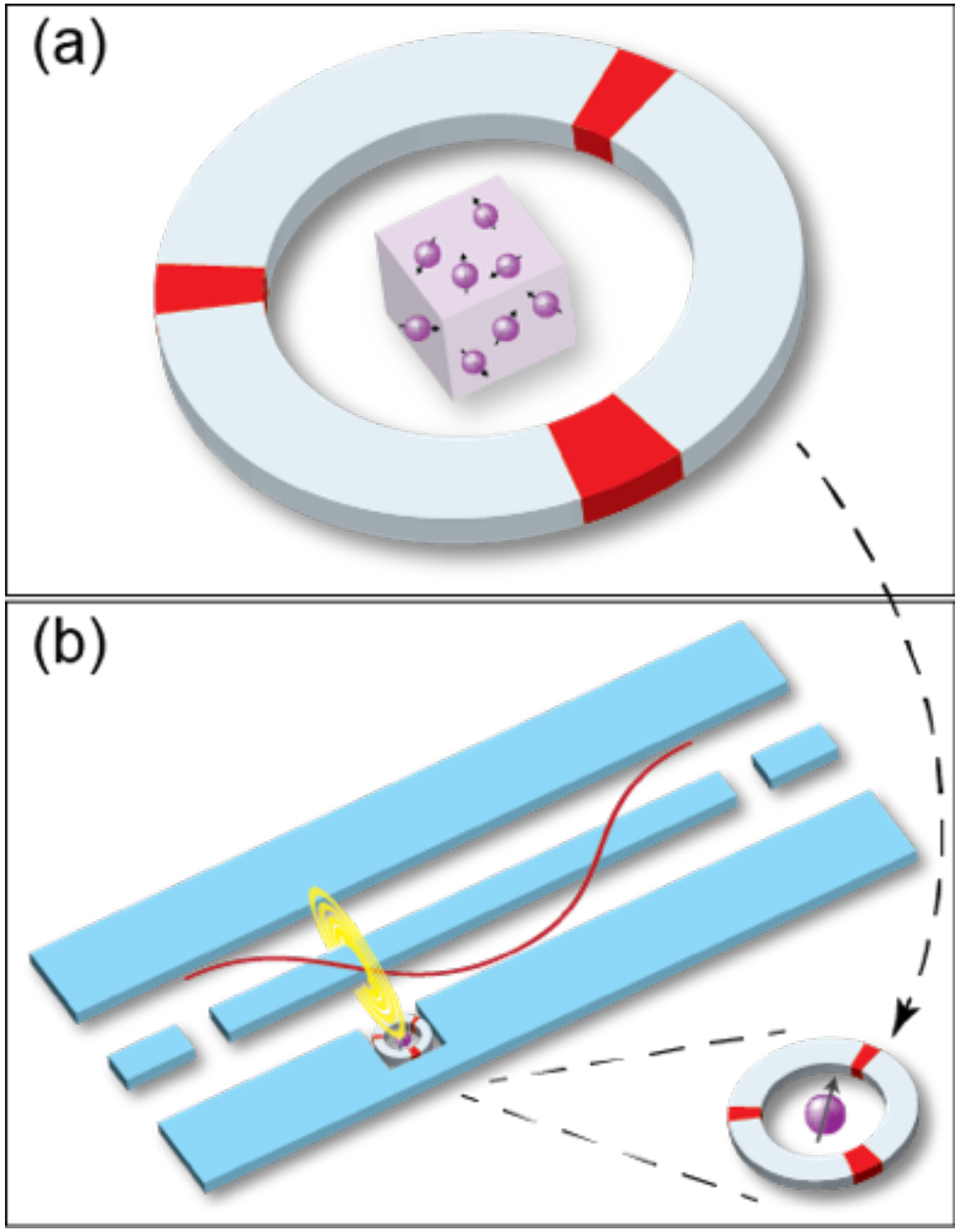


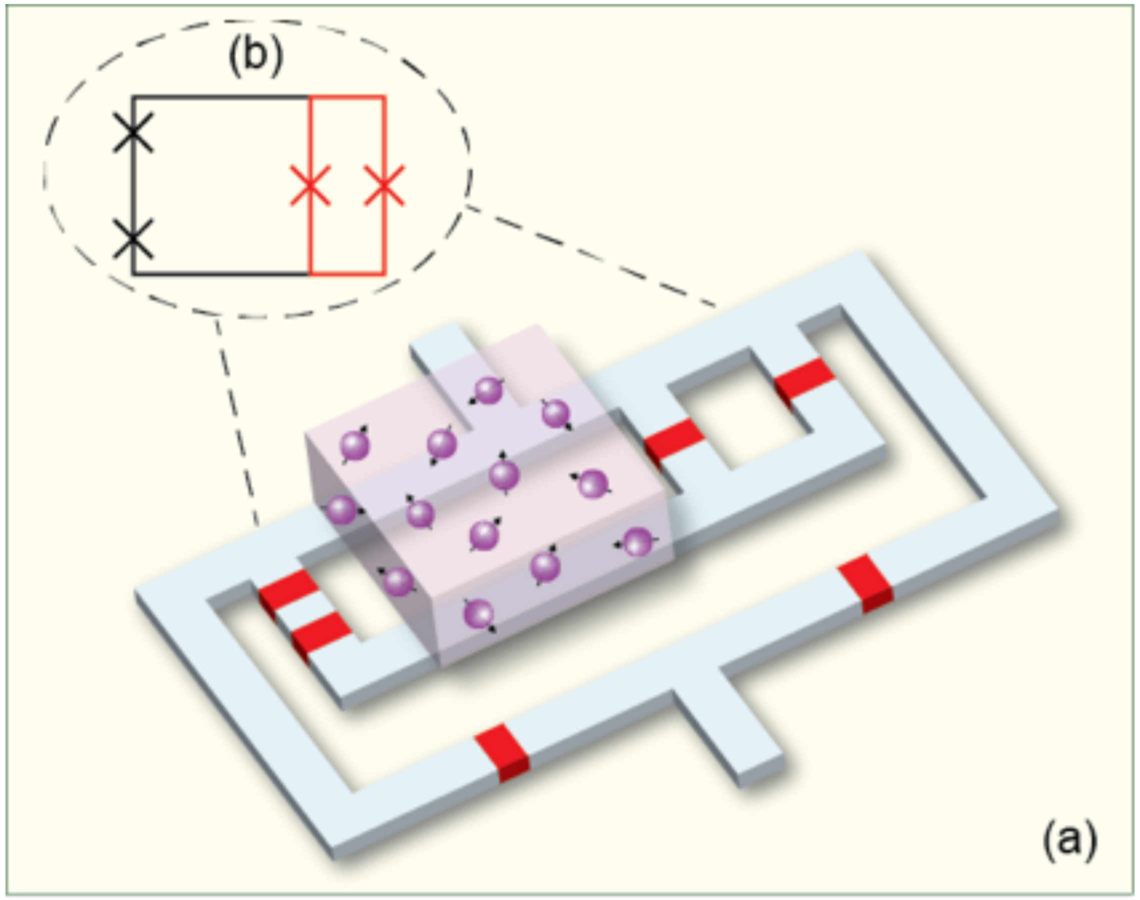


$$H = \hbar\omega_c a^\dagger a + \hbar\frac{\omega_c}{2}\sigma_a^z + \hbar\frac{\omega_s}{2}\sigma_s^z + \hbar g_{ca}(a^\dagger\sigma_a^- + a\sigma_a^+) + \hbar g_{cs}(a^\dagger\sigma_s^- + a\sigma_s^+)$$

theory: A.S. Sørensen et al., PRL (2004); L. Tian et al., PRL (2004); P. Rabl et al., PRL (2006)...
exp: Y. Kubo et al., PRL (2011); D. Schuster et al., PRL (2010), R. Amsüss et al., PRL (2011)…

Hybrid quantum circuits: Direct



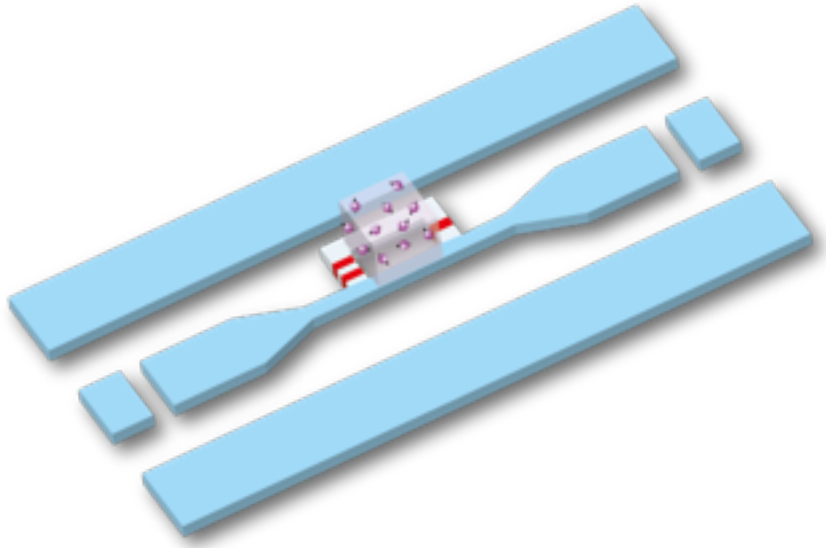


$$H = \hbar \frac{\omega_c}{2} \sigma_{nv}^z + \hbar \frac{\omega_s}{2} \sigma_s^z + \hbar g (\sigma_s^+ \sigma_{nv}^- + \sigma_s^- \sigma_{nv}^+)$$

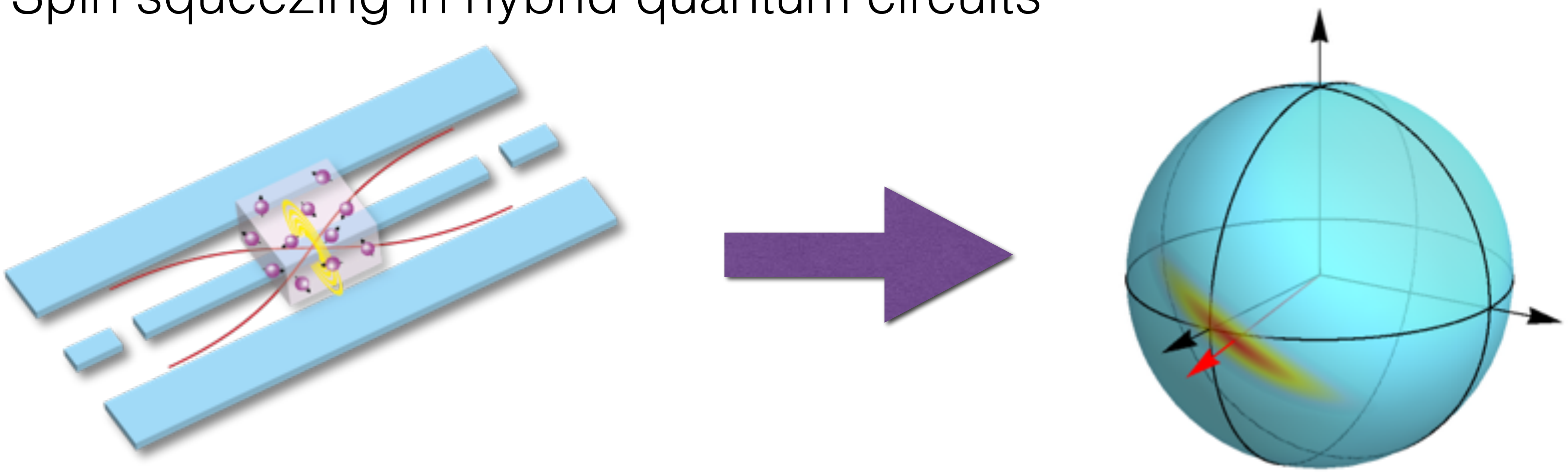
theory: D. Marcos et al. Phys. Rev. Lett. (2010); J. Twamley & S.D. Barrett, Phys. Rev. B (2012)
exp: X. Zhu et al. Nature (2011)

Outline

- Quantum information processing with hybrid quantum circuits



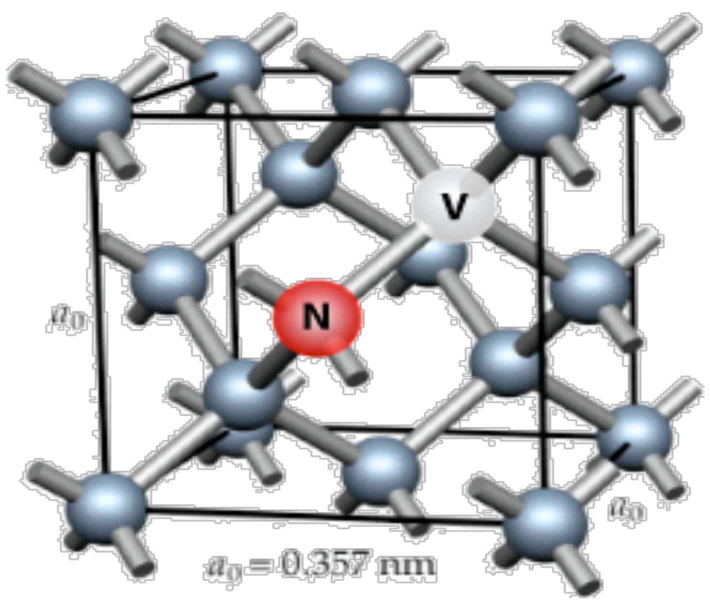
- Spin squeezing in hybrid quantum circuits

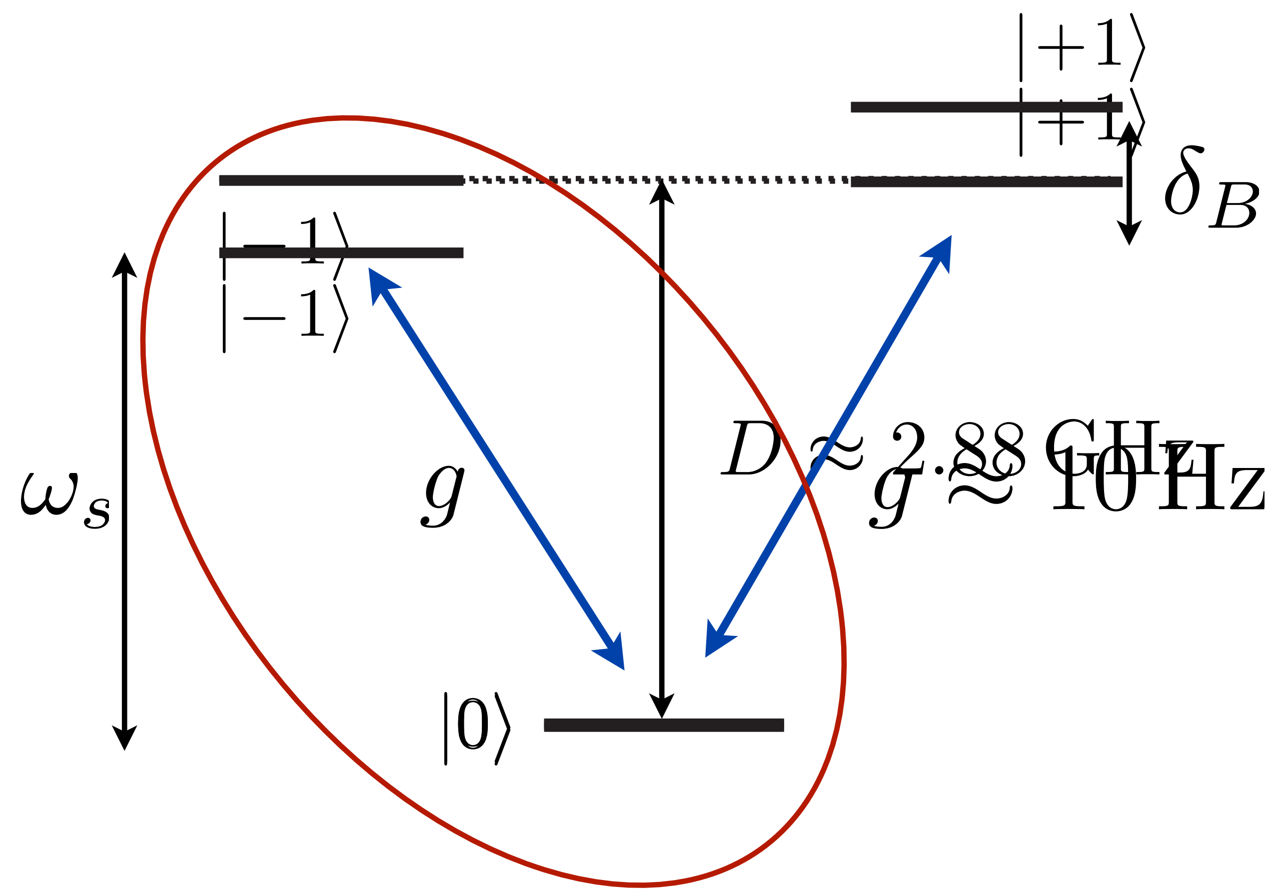


- Summary

NV center

NV - center



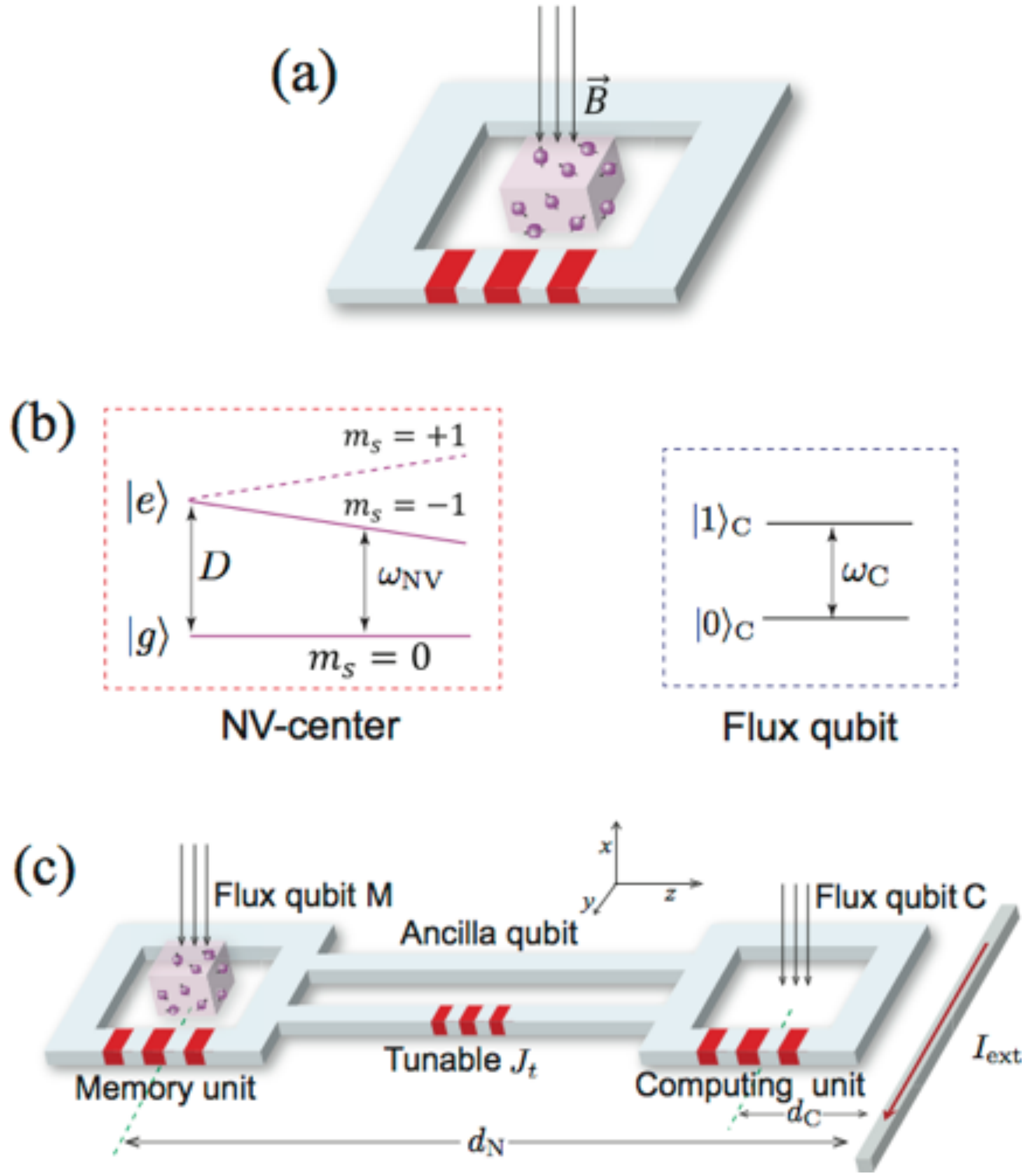


NV center (S=1 ground state):

$$H_{\mathrm{NV}} = \hbar D S_z^2 + \underbrace{\mu_B g_s B_{\mathrm{stat}}}_{\hbar\delta_B} S_z + \underbrace{\mu_B g_s B_{\mathrm{osc}}}_{\hbar g(a+a^\dagger)} S_x$$

Hanson, R. and D. D. Awschalom, Nature (2008)

Hybrid superconducting circuits with NV Centers I



Hamiltonian:

$$H_{\rm tot} = \sum_{j={\rm M,C}} \omega_j \tilde{\sigma}_j^+ \tilde{\sigma}_j^- + \omega_{\rm NV} b^\dagger b + g\left(\tilde{\sigma}_{\rm M}^+ b + b^\dagger \tilde{\sigma}_{\rm M}^-\right) + J_t\left(\tilde{\sigma}_{\rm M}^+ \tilde{\sigma}_{\rm C}^- + \tilde{\sigma}_{\rm C}^+ \tilde{\sigma}_{\rm M}^-\right)$$

Resonant interaction case:

$$H_{\rm tot}^R = g\left(\tilde{\sigma}_{\rm M}^+ b + b^\dagger \tilde{\sigma}_{\rm M}^-\right) + J_t\left(\tilde{\sigma}_{\rm M}^+ \tilde{\sigma}_{\rm C}^- + \tilde{\sigma}_{\rm C}^+ \tilde{\sigma}_{\rm M}^-\right)$$

Dispersive interaction case:

$$H_{\rm tot}^D = \Delta'_{\rm NV} b^\dagger b + \Delta'_{\rm C} \tilde{\sigma}_{\rm C}^\dagger \tilde{\sigma}_{\rm C}^- + \Lambda\left(\tilde{\sigma}_{\rm C}^- b^\dagger + \tilde{\sigma}_{\rm C}^\dagger b\right)$$

$$\Delta'_k = \Delta_k + \frac{g^2}{\Delta_k}$$

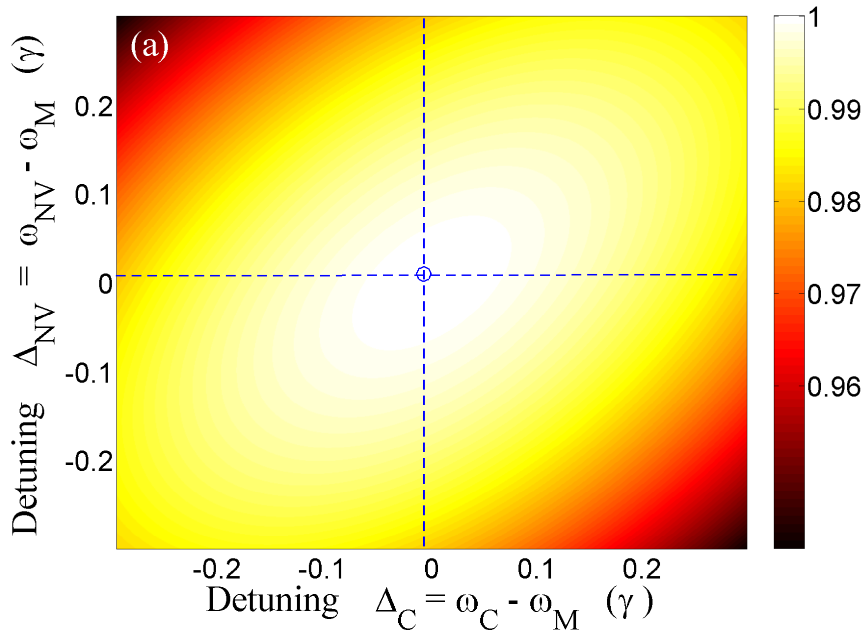
$$\Lambda = \frac{gJ_t}{2}\left(\frac{1}{\Delta_{\rm NV}} + \frac{1}{\Delta_{\rm C}}\right)$$

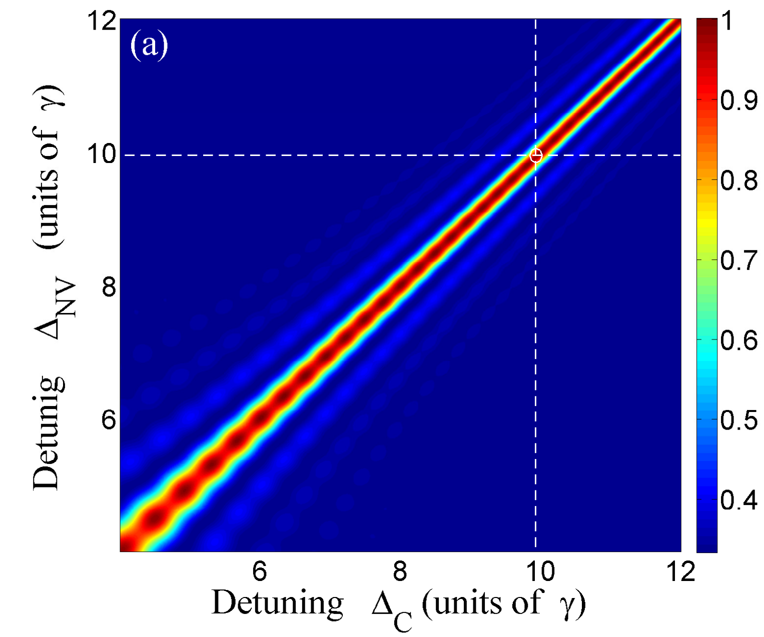
X. -Y. Lü, Z. -L. Xiang et al., Phys. Rev. A (2013)

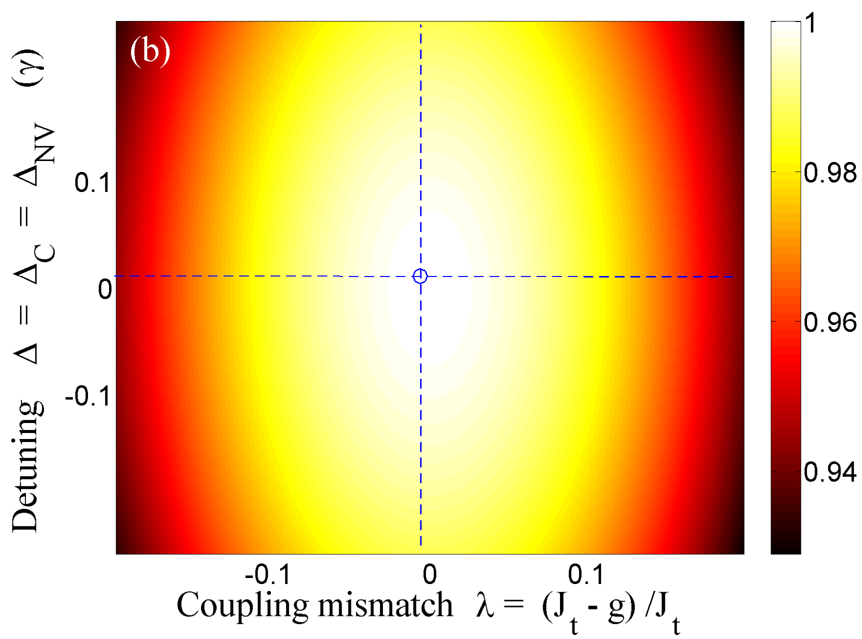
Fidelity in different interaction cases

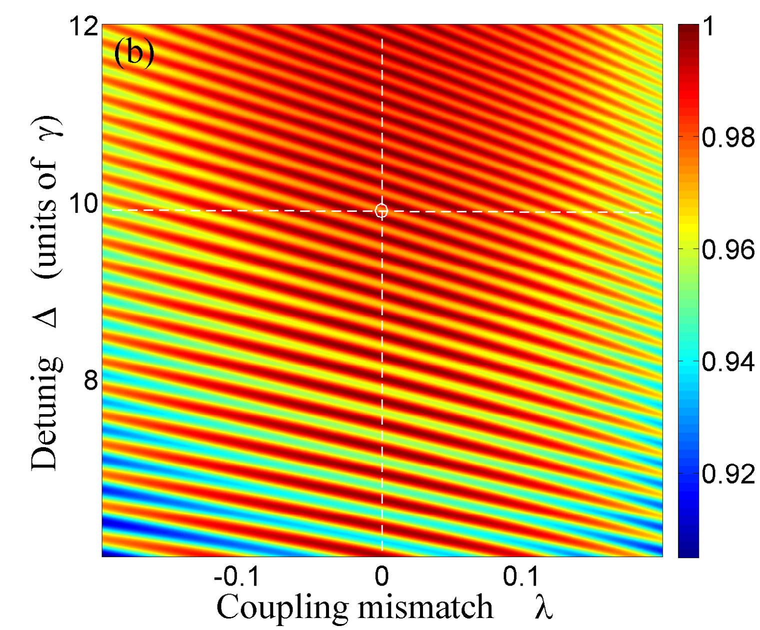
Fidelity is

$$|\langle \psi_T | \psi(t) \rangle|^2$$





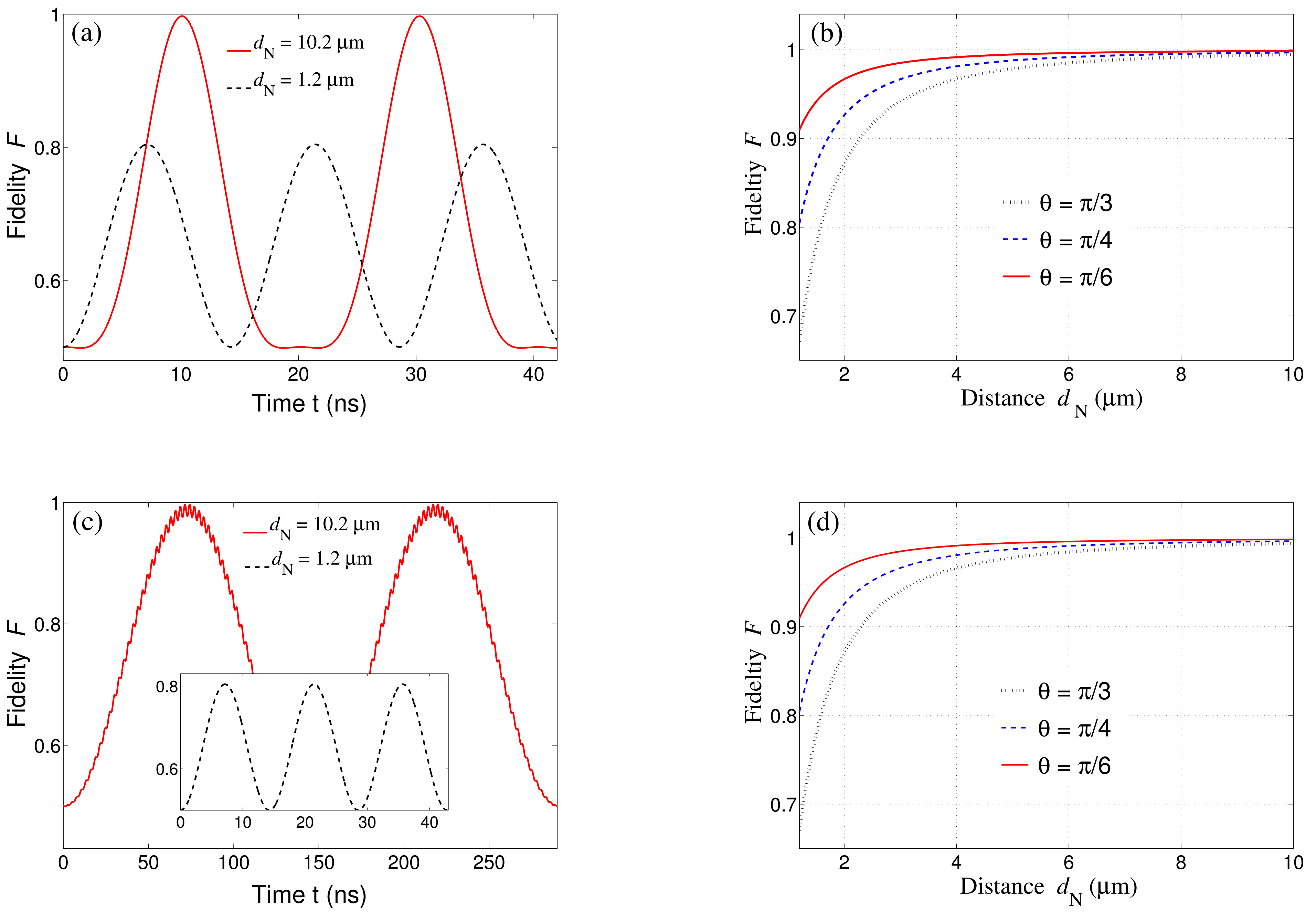




Resonant interaction case

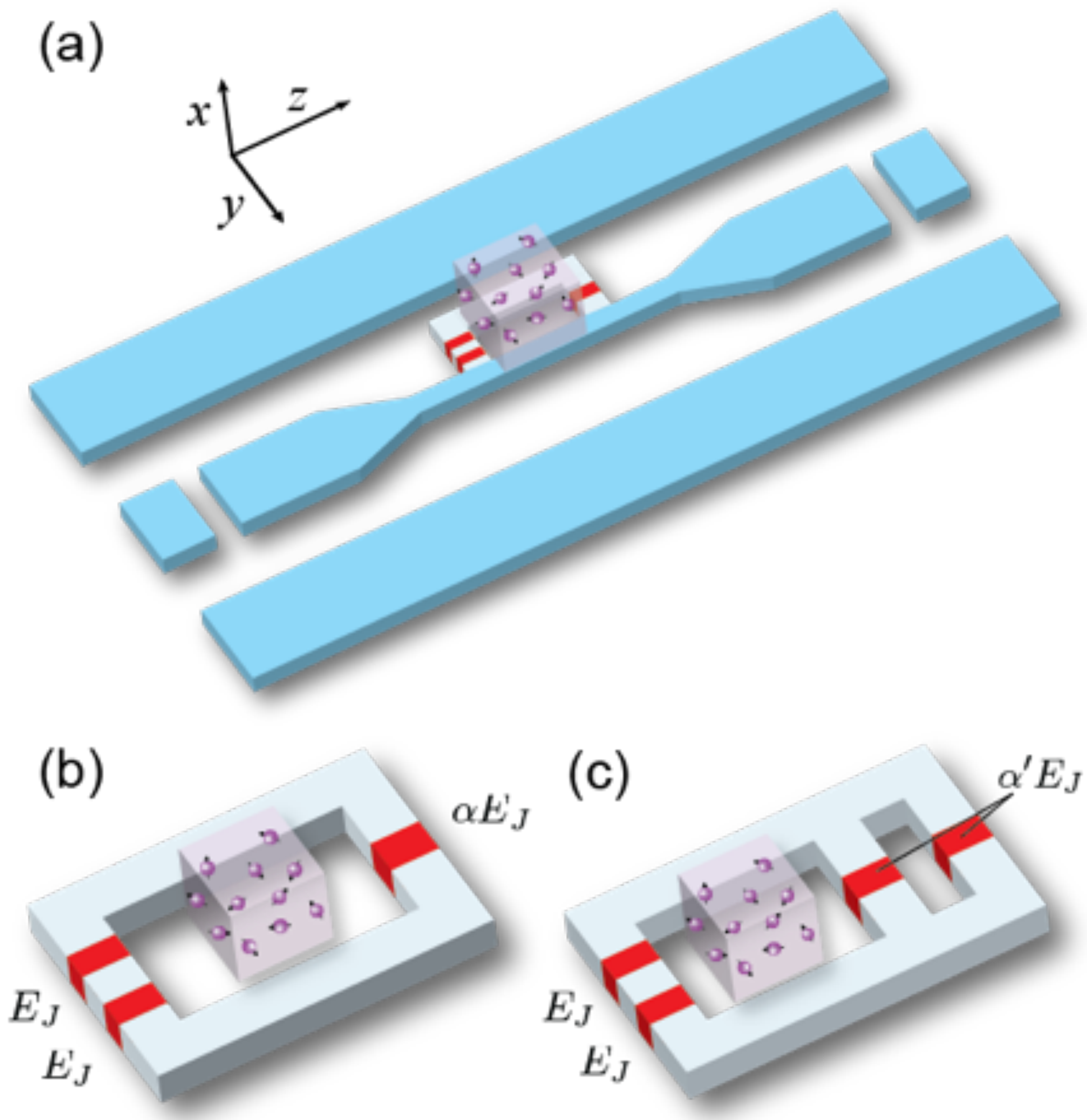
Dispersive interaction case

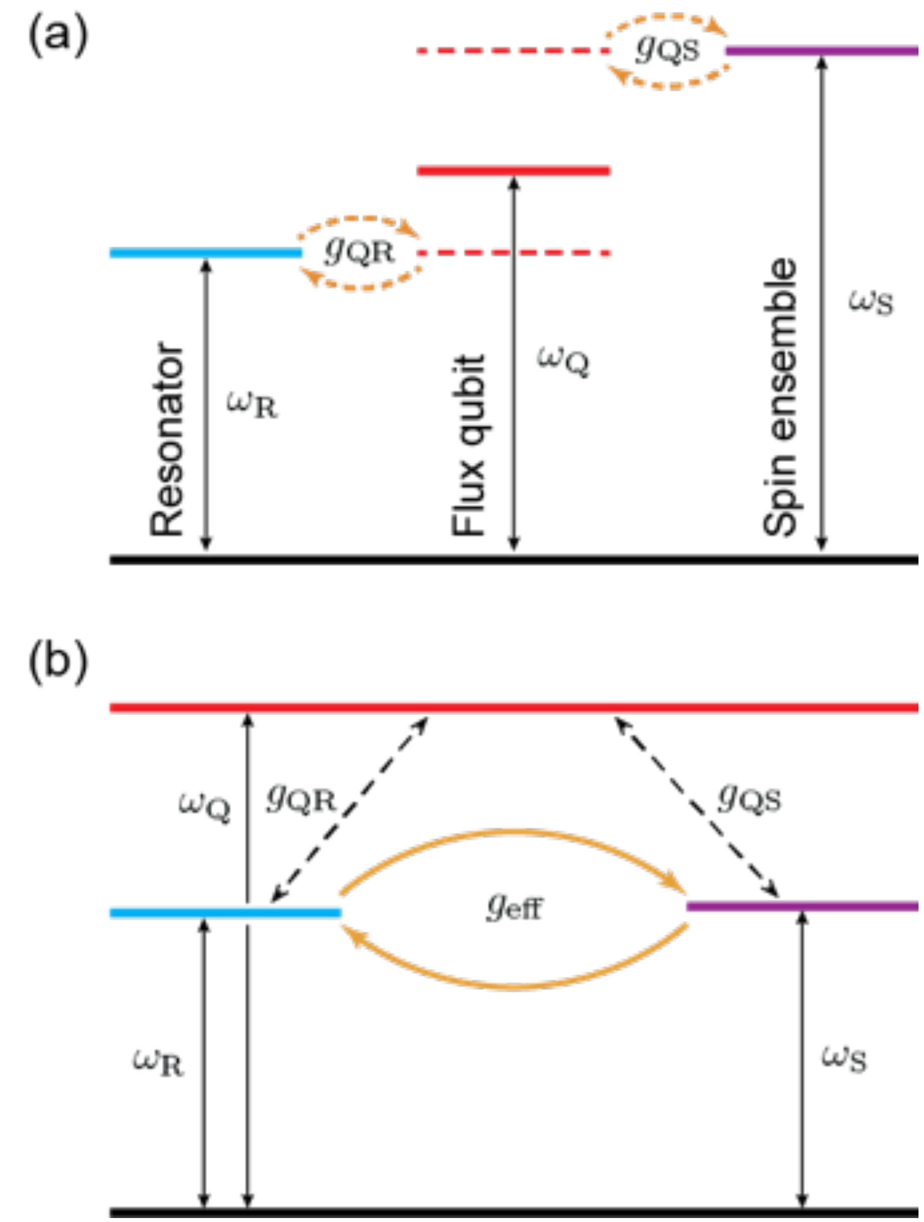
Fidelity with different distances



The fidelities of quantum storage versus (a,c) time t and (b,d) the distance. The black dashed and red solid curves in (a,c) correspond to the single flux-qubit–NVE system and the proposed system in this paper.

Hybrid superconducting circuits with NV centers II





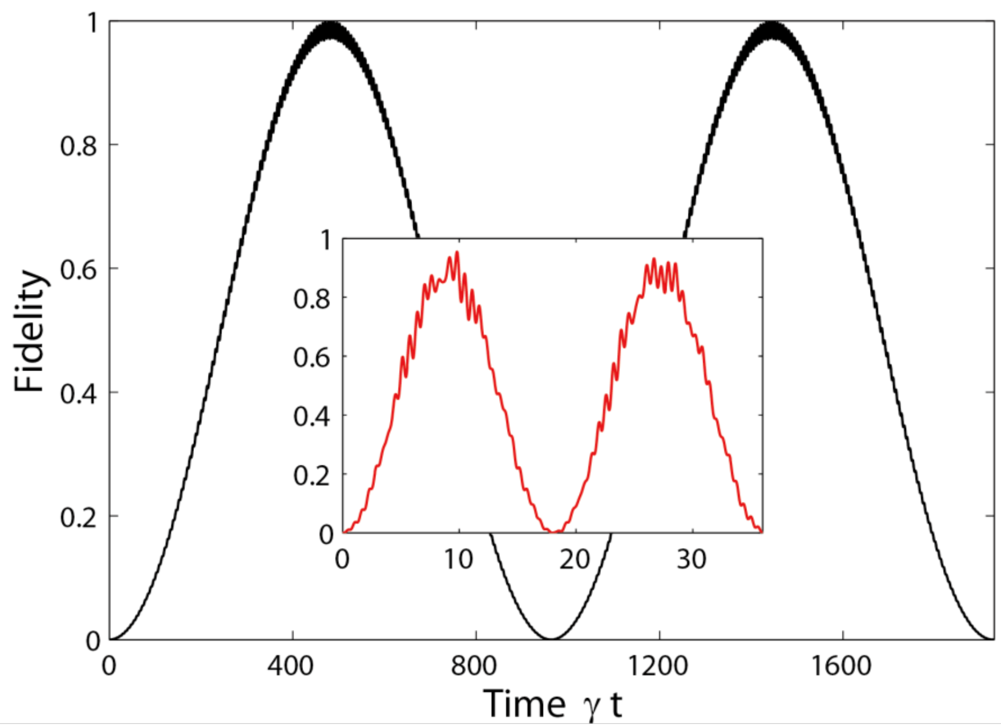
$$H = \frac{1}{2}\omega_q\sigma_z + \omega_r a^\dagger a + \omega_s s^\dagger s + g_{qr}(\sigma_+ a + \sigma_- a^\dagger) + g_{qs}(\sigma_+ s + \sigma_- s^\dagger)$$

$$H_{\mathrm{eff}} = \omega_r' a^\dagger a + \omega_s' s^\dagger s + g_{\mathrm{eff}}(a s^\dagger + a^\dagger s)$$

$$g_{\mathrm{eff}} = -\frac{1}{2}\left(\frac{1}{\Delta_r} + \frac{1}{\Delta_s}\right) g_{qr} g_{qs}$$

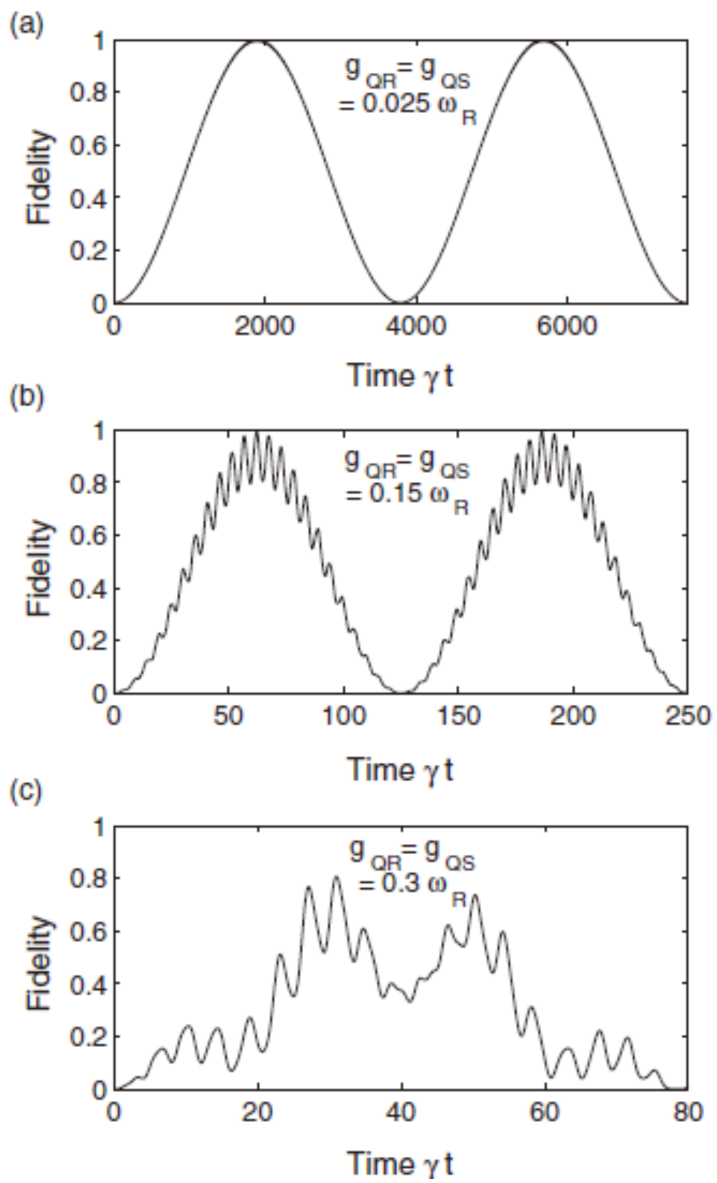
Z. -L. Xiang et al., Phys. Rev. B (2013)

Fidelity of quantum state transfer



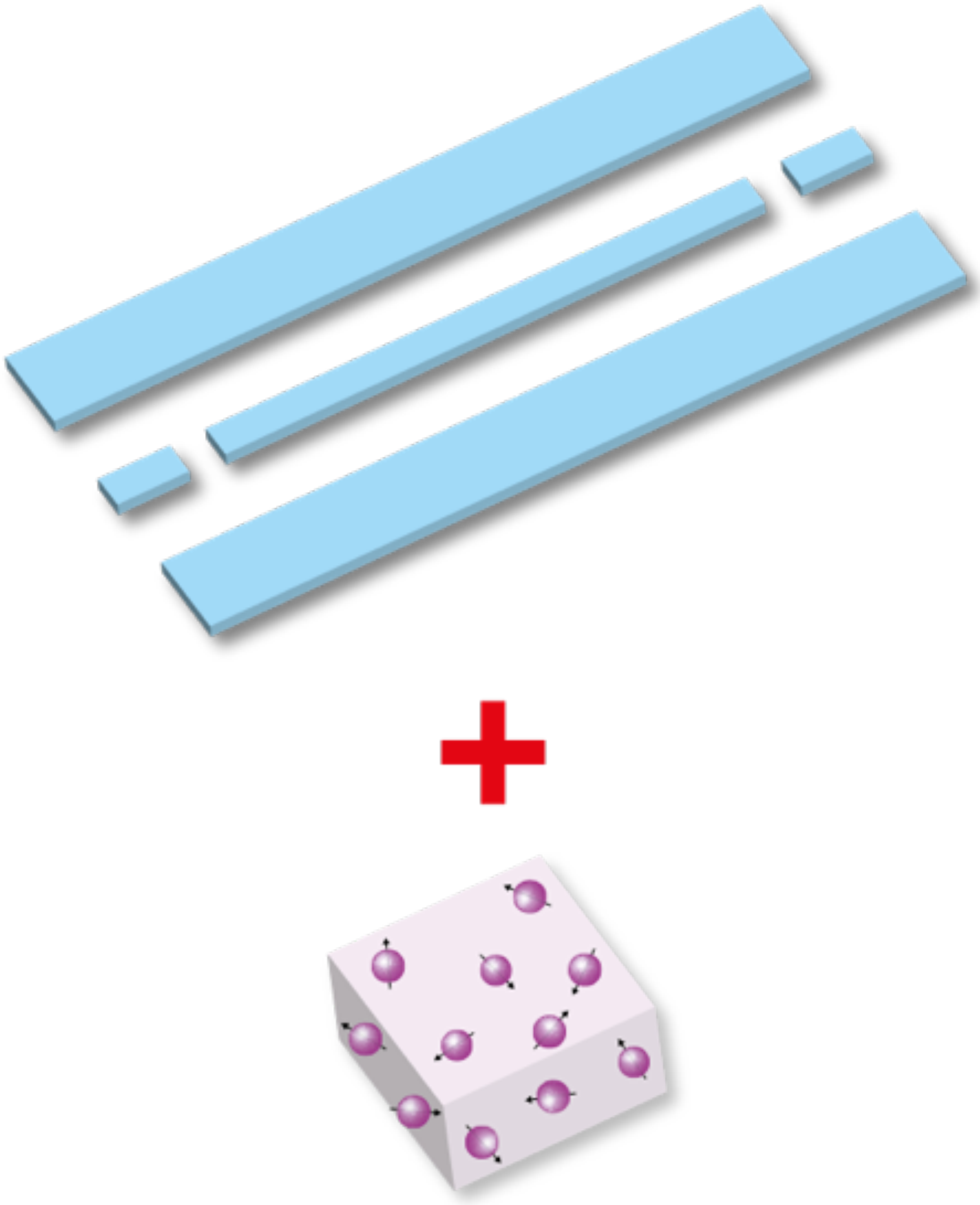
Fidelity is $|\langle\psi_T|\psi(t)\rangle|^2$

The fidelity of quantum state transfer vs the dimensionless time γ t . The red and black curves correspond to the coupling strength in the ultrastrong-coupling regime and the strong-coupling regime, respectively.



While the coupling strength is increasing, the fidelity is decreasing. After the large detuning condition is broken, the fidelity rapidly reduces to a low level, because high order terms in approximations cannot be neglected.

Can this hybrid circuit do anything else?

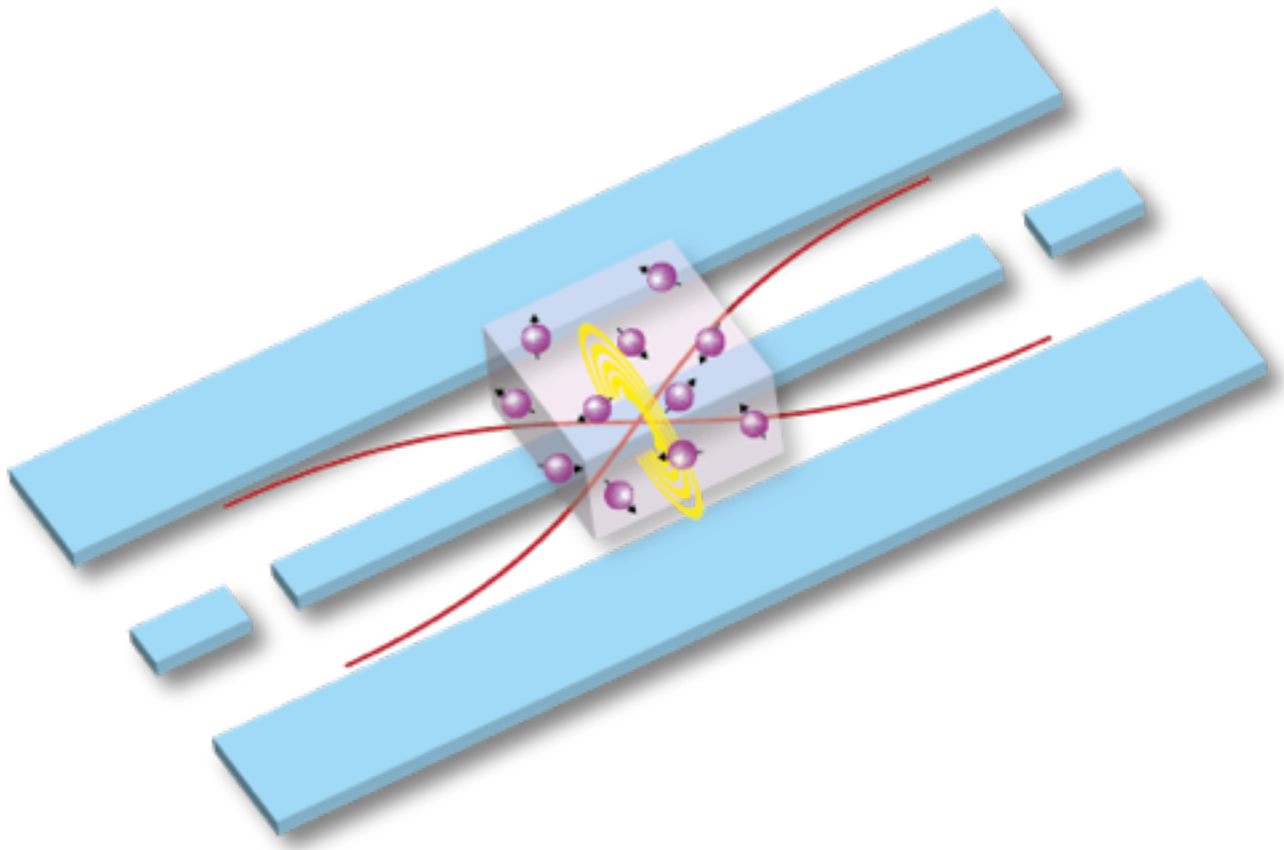


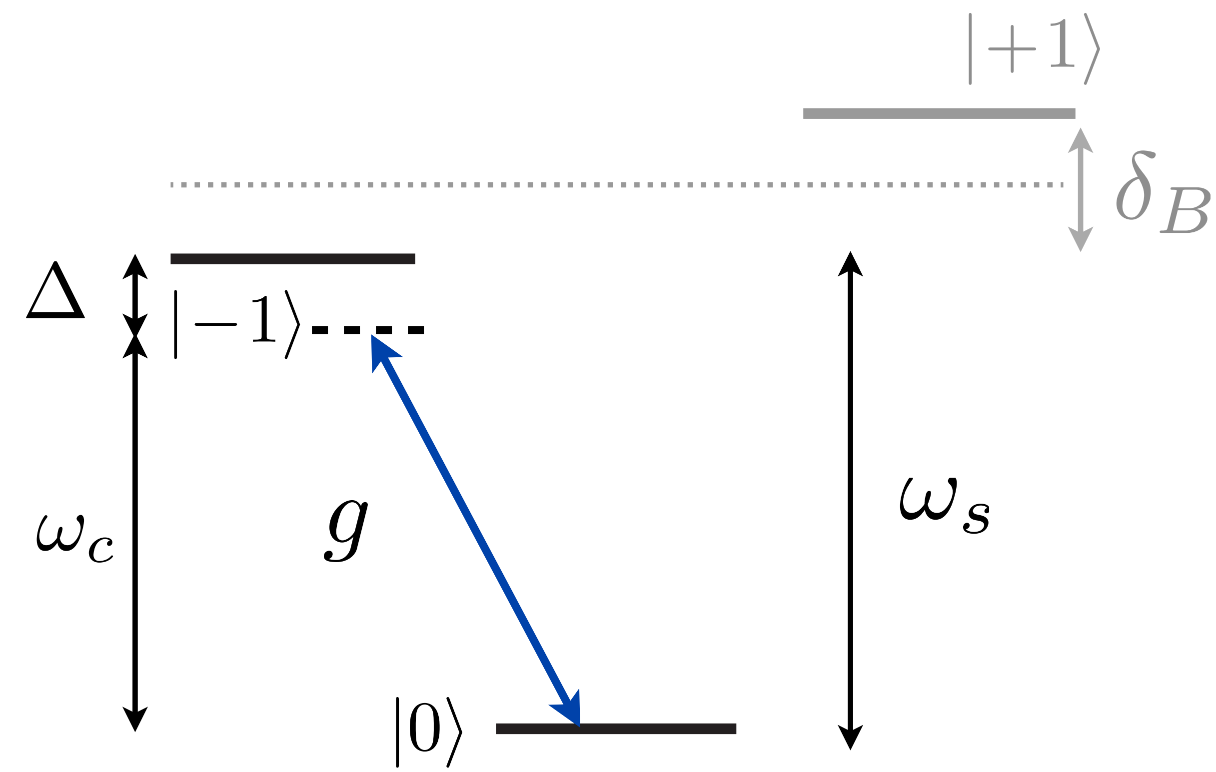
Quantum state transfer

Quantum storage
and memory

Squeezing!

Spin squeezing in hybrid quantum circuits





$$H = \omega_c a^\dagger a + \omega_s J_z + \frac{g}{\sqrt{N}}(J_+ a + a^\dagger J_-)$$

$$J_k = \frac{1}{2}\sum_{i=1}^{N}\sigma_k$$ collective spin operators

$$g = g_0\sqrt{N}$$ collective coupling

When $\Delta \gg g$, such a system can involve nonlinear property (λJ_z^2).

Mean-spin-direction (MSD) of the ensemble

Define

$$\alpha \equiv \langle a \rangle, \quad \beta \equiv \frac{\langle J_- \rangle}{N/2}, \quad \gamma \equiv \frac{\langle J_z \rangle}{N/2}$$

The master equation with the photon decay

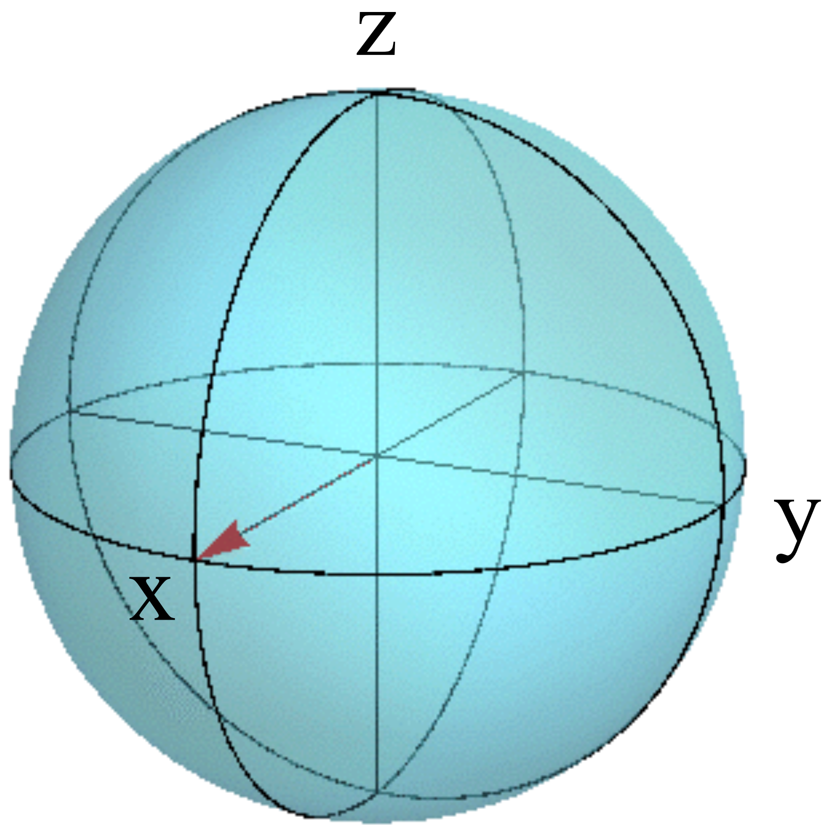
$$\dot{\rho} = -i[H, \rho] + \kappa(2a\rho a^\dagger - a^\dagger a\rho - \rho a^\dagger a)$$

With the mean value approximation, we can obtain the dynamical equations of MSD.

$$\dot{\alpha} = -(\kappa - i\Delta)\alpha - i\frac{g}{\sqrt{N}}\frac{N}{2}\beta,$$

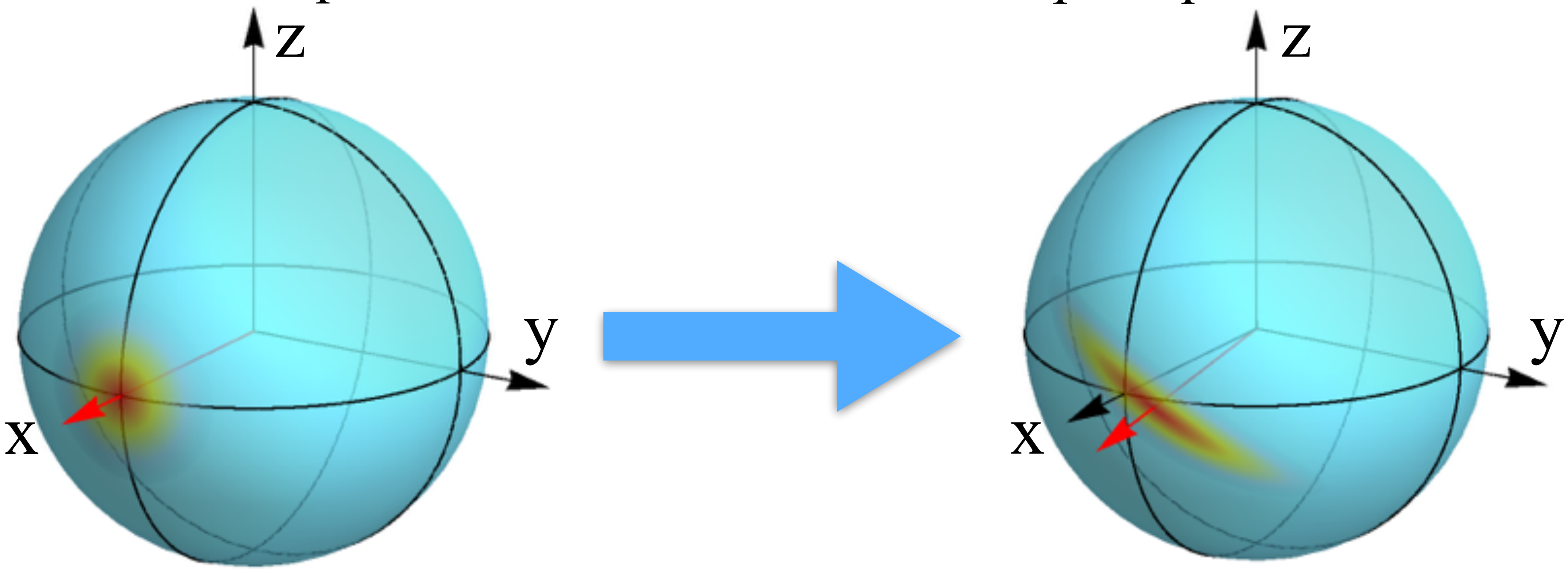
$$\dot{\beta} = 2i\frac{g}{\sqrt{N}}\alpha\gamma,$$

$$\dot{\gamma} = i\frac{g}{\sqrt{N}}(\alpha^*\beta - \alpha\beta^*).$$



Spin squeezing

Coherent spin states　　　　　Spin squeezed states



$$J_x|\psi_0\rangle = N/2|\psi_0\rangle = J|\psi_0\rangle$$

$$\langle J_y\rangle_0 = \langle J_z\rangle_0 = 0$$

$$\langle J_y^2\rangle_0 = \langle J_z^2\rangle_0 = J/2$$

$$\min\langle \Delta J_\perp^2\rangle < J/2$$

$$\langle \Delta J_1^2\rangle\langle \Delta J_2^2\rangle \geqslant J^2/4$$

Because of $|J| \gg 1$, we can make the Holstein-Primakoff transformation:

$$J'_+ = \sqrt{N}b, \quad J'_- = \sqrt{N}b^\dagger, \quad J'_z = \frac{N}{2} - b^\dagger b,$$

Spin squeezing with changing MSD

In the rotating frame of the MSD, we can obtain the effective Hamiltonian

$$H_{\mathrm{eff}} \approx -\frac{g^2}{N\Delta}(\sin^2\theta J_x'^2 + \cos^2\theta J_z'^2)$$
$$= -\frac{g^2}{4\Delta}\left[\sin^2\theta(b+b^\dagger)^2 + \cos^2\theta(N-2b^\dagger b)^2/N\right]$$

Here, $\theta(t)$ is time-dependent. By using the following master equation, we can numerically calculate the dynamics of this hybrid quantum system.

$$\dot{\rho} = -i[H_{\mathrm{eff}}, \rho] + \Gamma\left(2B^\dagger\rho B - BB^\dagger\rho - \rho BB^\dagger\right),$$

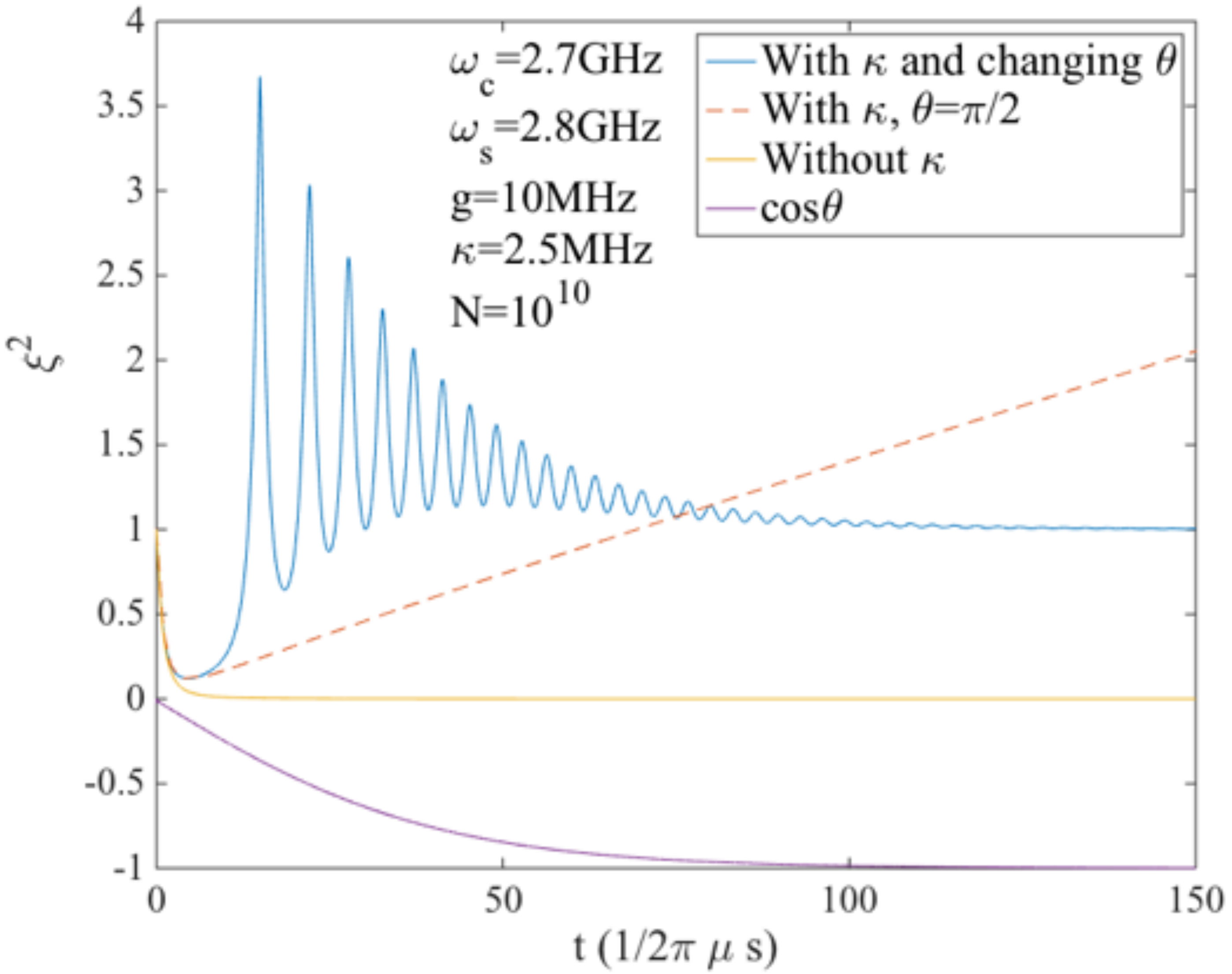
where $\Gamma = \frac{g^2}{\Delta^2}\kappa$ (the photon-induced decay),

$$B = \cos^2\frac{\theta}{2}b - \sin^2\frac{\theta}{2}b^\dagger.$$

Spin squeezing: ideal case

Spin squeezing parameter:

$$\xi^2 = \frac{N\Delta J^2_{\vec{n}_\perp}}{|\langle \vec{J} \rangle|^2} \approx 2\langle b^\dagger b\rangle - 2\sqrt{\langle b^2\rangle\langle b^{\dagger 2}\rangle} + 1$$



$$t_{\min} \approx \frac{\Delta}{g^2}\left[(3/\epsilon)^{\frac{1}{3}} - (\epsilon/3)^{\frac{1}{3}}\right]$$

$$\xi^2_{\min} \approx (3\epsilon^2)^{\frac{1}{3}} - (\epsilon^4/3)^{\frac{1}{3}} + O(\epsilon)^{\frac{8}{3}}$$

$$\epsilon = \kappa/\Delta$$

M. Kitagawa & M. Ueda, Phys. Rev. A (1993)

Spin squeezing: inhomogeneous broadening

Inhomogeneous broadening

$$\rightarrow t < T_2 \rightarrow$$

optimize over detuning.

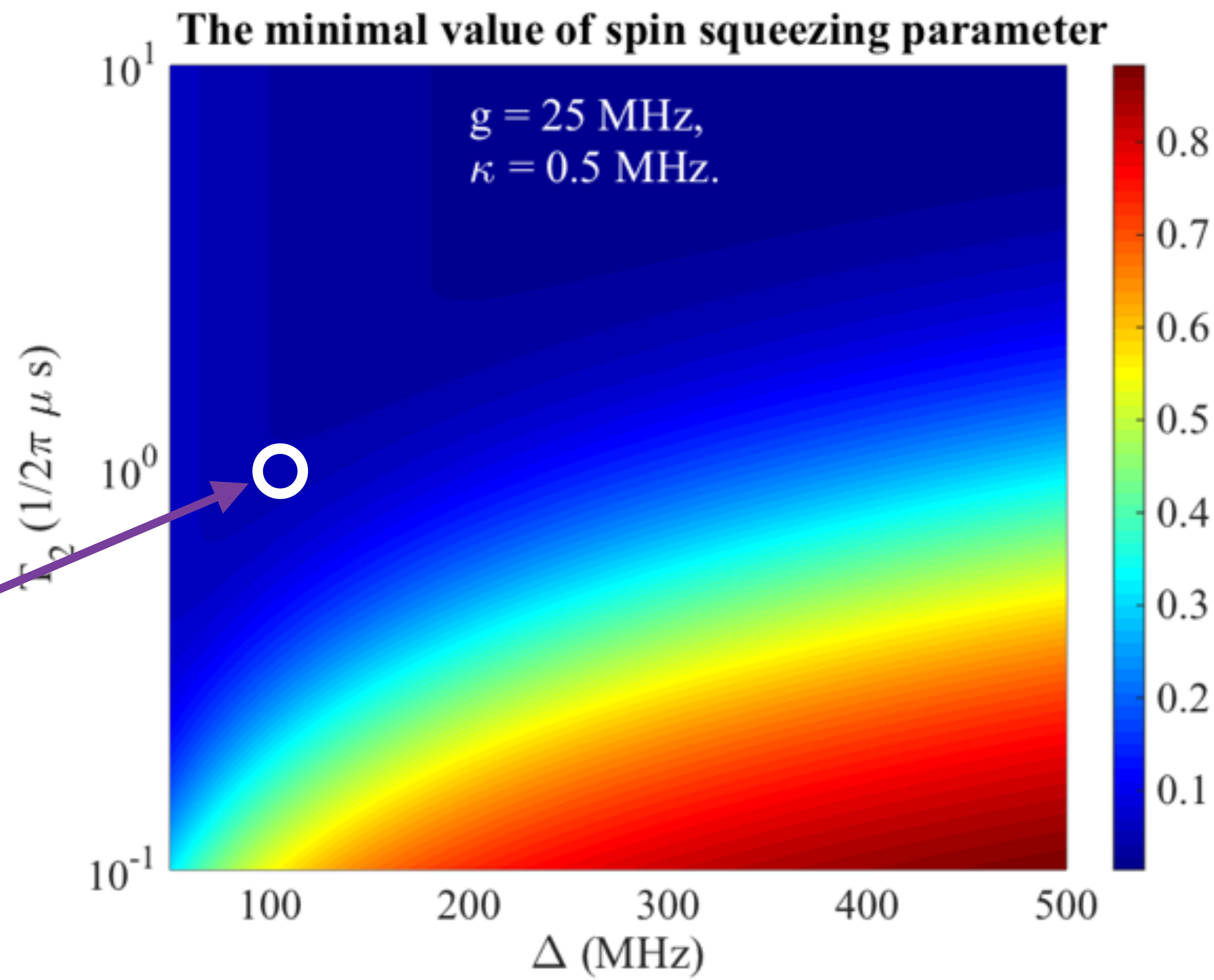
In experiment, $g, \ \kappa \rightarrow$ fixed,

$\Delta \rightarrow$ tunable.

$g = 25\text{MHz}, \quad \kappa = 0.25\text{MHz},$

$N \approx 10^{12}, \quad \Delta = 100\text{MHz},$

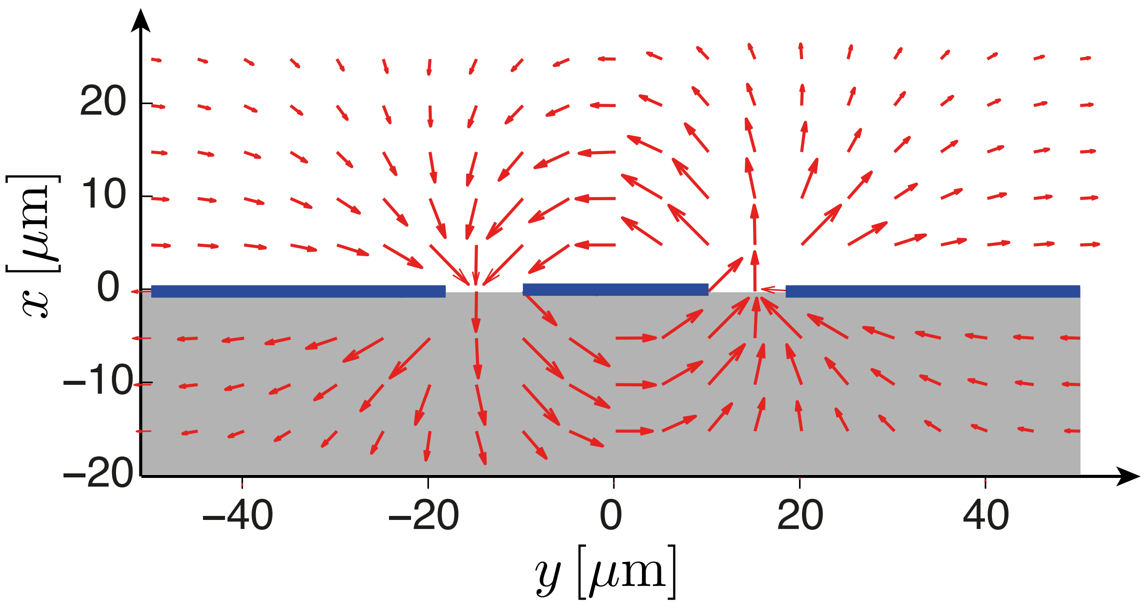
$T_2 = 1\mu\text{s}, \quad \xi^2 \approx 0.0347$

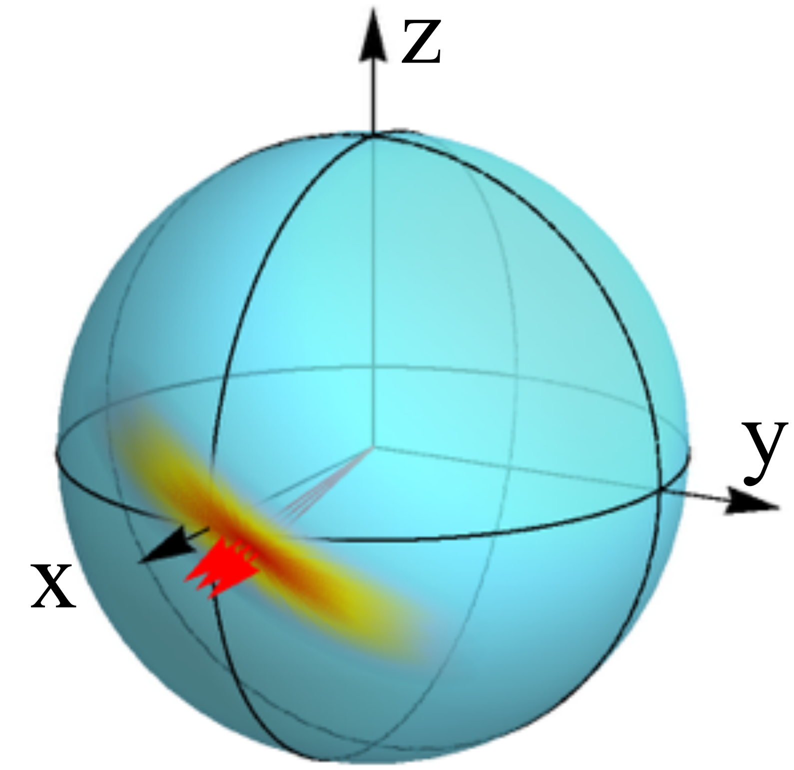


These parameters can be achieved by using current experimental techniques.

Y. Kubo et al., PRL (2010); R. Amsüss et al., PRL (2011); S. Putz et al., Nature Phys. (2014)

Spin squeezing of an inhomogeneous spin ensemble?





$$H = \omega_c a^\dagger a + \sum_\mu \omega_\mu J^z_\mu + \sum_\mu \frac{g_\mu}{\sqrt{N_\mu}} (J^+_\mu a + a^\dagger J^-_\mu).$$

Inhomogeneous frequencies

Inhomogeneous coupling strength

Spin squeezing ? In processing…

Summary

- Hybrid quantum circuits are to combine different systems and build new hybrid quantum structures for exploring new applications of quantum systems.
- Hybrid quantum circuits with NV centers and superconducting circuits can well perform in quantum storage and processing.
- The hybrid quantum circuits can also be used to explore the spin squeezing.

Thank you !