Schemes to generate entangled photon pairs via spontaneous parametric down conversion

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Outline

- Introduction
  - Optical parametric processes
    - Opt. param. amplifier (OPA)
    - Spontaneous param. down conv. (SPDC)
- Application | classical
  - **Broadband generation | for short pulse**
- Application | quantum
  - Entangled photon pairs
  - Ghost imaging | wave vector
  - **Ghost spectroscopy | frequency**
  - Quantum key distribution (QKD) | polarization
  - **Multiplex QKD | polarization and frequency**
  - Entangled photon beam
- Conclusion
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Outline

- Introduction | OPA and SPDC
  - Light is …
  - Light-matter interaction
  - Frequency conversion
    - SHG and SPDC
    - SPDC ⇔ OPA
    - OPA is …?
  - Why OPA?
  - Why SPDC?

![Image of experimental setup]
Introduction

Light

- gamma-ray
- X-ray
- Ultraviolet
- visible
- infrared
- radio wave

=> Electro-magnetic wave
Introduction

Light-matter interaction
Electric field make dielectric polarization

\[ P = \varepsilon_0 \left( \chi^{(1)} E + \chi^{(2)} EE + \chi^{(3)} EEE \cdots \right) \]

Emission from dipole
oscillating in vertical direction
\[ E : \exp(-i\omega t) \]
\[ E^*E : \exp(-i2\omega t) \]

Second harmonic generation (SHG)
using a non-linear crystal
within some limitation from physical law…
Introduction

energy / momentum conservation in frequency mixing

\[ \hbar \omega_{SHG1} = \hbar \omega_1 + \hbar \omega_1 \]
\[ \hbar \omega_{SHG2} = \hbar \omega_2 + \hbar \omega_2 \]
\[ \hbar \omega_{SFG} = \hbar \omega_1 + \hbar \omega_2 \]

\[ k_{SHG1} = k_1 + k_1 \]
\[ k_{SHG2} = k_2 + k_2 \]
\[ k_{SFG} = k_1 + k_2 \]
Introduction

SHG (second harmonic generation)
\[ \omega \rightarrow 2\omega \]

Reversible? YES!

Reverse process
spontaneous parametric down conversion (SPDC)

occur by itself

\[ 2\omega \rightarrow \omega + \omega \]
\[ 2\omega \rightarrow 0.8\omega + 1.2\omega \]
Introduction

How does SPDC occur? similar as OPA (optical parametric amplification)

\[ h\nu_{\text{pump}} - h\nu_{\text{signal}} = h\nu_{\text{idler}} \]

Energy conservation
\[ h\nu_{\text{pump}} = h\nu_{\text{signal}} + h\nu_{\text{idler}} \]

\#signal=\#idler

SPDC starts with vacuum noise (no seed for signal)

quite low efficiency \(~10^{-10}\)
Introduction

Why SPDC? low conversion efficiency 
interesting character of entanglement 
never broken security | quantum communication 
easy to transfer | via optical fiber 
Other method?
singlet (a pair of spin $\frac{1}{2}$ particle)
**Introduction**

**OPA** (optical parametric amplification)  
...what is **OPA**?  
similar as SPDC, much higher efficiency

\[ h_{\text{pump}} - h_{\text{signal}} = h_{\text{idler}} \]  
Energy conservation  
\[ h_{\text{pump}} = h_{\text{signal}} + h_{\text{idler}} \]
Introduction

Why OPA? complicated setup

*intense laser at different* wavelength

*non-linear spectroscopy* in UV/visible/IR…

(ultrafast spectroscopy, Raman for vibration study, …)

Other method?

self phase modulation (SPM) | low efficiency
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Application | classical

Broadband generation | for short pulse

\[ E_y = \text{Re} \left( E_0 e^{i\omega_0 t} \right) \]
\[ E_y = \text{Re} \left( E_0 e^{(-\Gamma t^2 + i\omega_0 t)} \right) \]

Shorter pulse needs broader spectrum

\[ \propto \frac{1}{\sqrt{\Gamma}} \]
\[ \propto \sqrt{\Gamma} \]

\[ E(\omega) = \exp \left[ \frac{-(\omega - \omega_0)^2}{4 \Gamma} \right] \]
Application | classical

Broadband generation | for short pulse

Optical parametric amplifier (OPA) Non-collinear OPA (NOPA)
Application | classical

Broadband generation | for short pulse

WLC

pulse width=\sim9\text{fs}

OPA (OPG with WLC)

Spectrum diffracted by grating

Visible broadband
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Application | quantum

SPDC generates photon pairs (low efficiency)

\[ \omega_p, k_p \rightarrow \omega_s, k_s \quad \text{NLC} \rightarrow \omega_i, k_i \]

correlated parameters

(1) wave vector: \( k_p = k_s + k_i \)
(2) frequency: \( \omega_p = \omega_s + \omega_i \)
(3) polarization: \( \frac{1}{\sqrt{2}} \left[ \langle H \rangle_s \mid V \rangle_i + \langle V \rangle_s \mid H \rangle_i \right] \)
(in case of Type-II crystal)
Application | quantum

So, what is entanglement?
Let’s remind “Young’s double slit” photon comes one by one if you block one of the slits…

Interference only in unknown case path entanglement

Interference of “probability”, “wavefunction” different from statistics of classical phenomena=quantum

Are there any other entanglements?
Yes, we will see them in the following pages!
Application | quantum

quantum lithography | wave vector

better resolution (than classical limit) \( \sim \lambda_p = \lambda / 2 \)

Schematic set-up

\[
E_1^{(+)} = a_s \exp(i k r_{A1}) + b_s \exp(i k r_{B1}) \\
E_2^{(+)} = a_i \exp(i k r_{A2}) + b_i \exp(i k r_{B2}) \\
R_c \propto P_{12} = \varepsilon^2 |\exp(i k r_A + i\varphi_A) + \exp(i k r_B + i\varphi_B)|^2 \\
\propto 1 + \cos[k (r_A - r_B)] \\
R_c(x) \propto \text{sinc}^2[(2\pi a/\lambda)\theta] \cos^2[(2\pi b/2\lambda)\theta] \\
\text{(Young’s: } \cos^2[(2\pi b/2\lambda)\theta] )
\]

“SPDC photon pairs” v.s. “classical light”
Experimental result (quantum lithograph)

Figure 8. (a) Experimental measurement of the coincidences for the two-photon double-slit interference-diffraction pattern. (b) Measurement of the interference-diffraction pattern for classical light in the same experimental setup. With respect to the classical case, the two-photon pattern has a faster spatial interference modulation and a narrower diffraction pattern width, by a factor of 2.
Application | quantum

ghost imaging | wave vector
Application | quantum

ghost imaging | wave vector

![Diagram of quantum ghost imaging with BBO crystals and coincidence counts for CC1, CC2, and CC3.]
**Application | quantum**

**ghost imaging**

measure the shape of an object

*Detector does NOT scan after object*

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**Diagram**

- Laser pump
- BBO
- Prism
- Collection lens
- Aperture
- Filter
- Gated N₂
- X-Y scanning fiber
- NLC
- ωᵢ, kᵢ
- ωₛ, kₛ
- ωₚ, kₚ
Application | quantum

ghost spectroscopy | frequency
Application | quantum

ghost spectroscopy | frequency
BBO: non-linear crystal
M1: parabolic mirror
M2,3: plane mirror
P1: prism (remove pump)
P2: prism (compensate angular dispersion)
PBS: polarizing beam splitter
G: diffraction grating
L2,3: fiber coupling lens
OF: optical fiber
SPCM: single photon counting module
TAC: time-to-amplitude converter
Delay: delay module
PC: computer
S: sample (Nd³⁺-doped glass)
L1: focusing lens
(f=100mm, 8mm)
Spectrum of photon pairs and absorption spectrum of the sample

pump focusing lens (f=100mm)

more absorption in longer wavelength
result: absorption spectrum

calculate absorption spectrum from the ratio
→ agree with the result by a spectrometer
result: absorption spectrum

1. Broadband photon pairs

agree with the result by a spectrometer
summary of this section

- Spectrum of SPDC photon pairs
  - Spherical lens → objective lens (f=100 → 8mm)
  - Spectrum was broadened (11,11 → 63,69nm)
- Nd$^{3+}$-doped glass (in the idler light path)
  - Coincidence resolving signal light’s frequency
  - Absorption spectrum was measured
    - Fit well with the result measured by a spectrometer
    - Without resolving the frequency of photon transmitted through the sample

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Our work
Outline for “Quantum Key Distribution (QKD)”

- BB84 protocol | single photon
  - how it works
  - can it be safe?
- E91 protocol | polarization entangled photon pair
  - polarization entanglement?
  - how it works
  - can it be safe?
**Application | quantum**

- **BB84 protocol | single photon**
  - Purpose: to share a secret key
  - how it works?
    - key at random
    - base at random

```plaintext
  key at random 0   1   0   1   1   0
  base at random +   +   +   +   +   +
```

```plaintext
  base at random ×   ×   +   +   ×   +
```

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**Application | quantum**

- **BB84 protocol | single photon**
  - **Purpose**: to share a secret key
  - **how it works?**
    - key at random
    - base at random
  - \[ \begin{array}{ccc}
  1 & 0 & 0 \\
  \times & + & + \\
  \end{array} \]
  - base at random
  - \[ \begin{array}{ccc}
  0 & 1 & 1 \\
  \times & + & + \\
  \end{array} \]

50\% of keys can be shared (shared keys are same)

complicated…**But secure!**

*How can it be secure?*
Application | quantum

BB84 protocol | single photon

Can it be secure?

- key at random 0 1 0 1 1 0
- base at random + × + × + +

base? (random try) + × × × + + ×

result (bit) 0 1 1 1 1 1 0

copy 0 1 1 1 0 1 1

base at random × × + + × +
**Application | quantum**

- BB84 protocol | single photon
  - Can it be secure?
    - key at random
    - base at random

Security can be checked!

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base? (random try)

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<th>0</th>
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<tr>
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<td>×</td>
<td>+</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
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Error!
polarization-entangled photon pairs

EPR-Bell source

Alice

Bob

$$|\Psi_{12}\rangle = \frac{1}{\sqrt{2}} \left[ |\uparrow \downarrow\rangle_1 |\leftrightarrow\rangle_2 - |\leftrightarrow\rangle_1 |\uparrow \downarrow\rangle_2 \right]$$

$$= \frac{1}{\sqrt{2}} \left[ |\rightarrow \rightarrow\rangle_1 |\nearrow \nearrow\rangle_2 - |\nearrow \nearrow\rangle_1 |\rightarrow \rightarrow\rangle_2 \right]$$

$$|\uparrow \downarrow\rangle = \frac{1}{\sqrt{2}} \left( |\rightarrow \nearrow\rangle + |\rightarrow \overleftarrow{\downarrow}\rangle \right)$$

$$|\leftrightarrow\rangle = \frac{1}{\sqrt{2}} \left( |\rightarrow \rightarrow\rangle - |\leftarrow \leftarrow\rangle \right)$$
1. Broadband photon pairs

Mixed state (statistical mixture)

HV and VH (50%-50%)

Alice

Bob
QKD example (without Eve)

If they use the same base, "100%" correlation
(quantum key distributed!)
QKD example (with Eve)

Eve also share the key (NOT secure QKD…)
How can it be improved?
Ekert91 protocol

Alice Base select EPR-pair Base select Bob

Base information (classical communication)

“100%” correlation

0 H + 0 V 0
0 R 0 L 0
1 L 1 R 1
1 V + 1 H 1
Ekert91 protocol

Base information

Bob can detect Eve (secure!)

OK

NG!
FIG. 2. The polarization entangled photons are transmitted via optical fibers to Alice and Bob, who are separated by 360 m, and both photons are analyzed, detected, and registered independently. After a measurement run the keys are established by Alice and Bob through classical communication over a standard computer network.

T. Jennewein et al., PRL 84, 4729 (2000)

FIG. 3 (color). The 49984 bit large keys generated by the BB84 scheme are used to securely transmit an image [23] (a) of the “Venus von Willendorf” [24] effigy. Alice encrypts the image via bitwise XOR operation with her key and transmits the encrypted image (b) to Bob via the computer network. Bob decrypts the image with his key, resulting in (c) which shows only a few errors due to the remaining bit errors in the keys.
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Generation of photon pairs entangled in their frequencies and polarizations (for WDM-QKD)

BBO (type-II)

frequency-entangled

ω - δω
ω + δω

polarization-entangled pair

at many wavelength combinations

light source for WDM-QKD
Standard:

Multiplex:
**Experimental Setup**

- **L1**: focusing lens
- **BBO**: non-linear crystal
- **M1**: parabolic mirror
- **M2,3**: plane mirror
- **P1**: prism (remove pump)
- **P2**: prism (compensate angular dispersion)
- **G**: diffraction grating
- **L2,3**: fiber coupling lens
- **OF**: optical fiber
- **SPCM**: single photon counting module
- **TAC**: time-to-amplitude converter
- **Delay**: delay module
- **PC**: computer
- **IRIS**: iris diaphragms
- **BS**: non-polarizing beam splitter
- **POL1,2**: linear polarizer
1. Broadband photon pairs

\[ |\psi\rangle = |H\rangle_s |V\rangle_i + f \cdot e^{i\alpha} |V\rangle_s |H\rangle_i \]

\( f=1 \)
\( \alpha=0^\circ \)

\( f=1 \)
\( \alpha=60^\circ \)

\( f=1 \)
\( \alpha=180^\circ \)

\( f=1.732 \)
\( \alpha=0^\circ \)
**polarization correlation** (1st diffraction@870nm)

\[
\left|\psi\right\rangle = \left|H\right\rangle_s \left|V\right\rangle_i + f \cdot e^{i\alpha} \left|V\right\rangle_s \left|H\right\rangle_i
\]

- **phase shift (866nm)**
  - < phase shift (870nm)
  
  \[f = 1.7 \rightarrow 1\]

- **visibility < 100%**
  
  \[\alpha \neq 0, 180^o\]

1. Broadband photon pairs
polarization correlation (1\textsuperscript{st} diffraction@870nm)

\[ |\psi\rangle = |H\rangle_s |V\rangle_i + f \cdot e^{i\alpha} |V\rangle_s |H\rangle_i \]

<table>
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<th>visibility</th>
<th>relative phase</th>
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<tr>
<td>0\textdegree</td>
<td>0.75</td>
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<tr>
<td>45\textdegree</td>
<td>0.43, -25\textdegree</td>
</tr>
<tr>
<td>135\textdegree</td>
<td>0.31, 35\textdegree</td>
</tr>
<tr>
<td>90\textdegree</td>
<td>0.50, -81\textdegree</td>
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</table>

phase shift (866nm) < phase shift (870nm) \( f = 1.7 \rightarrow 1 \)

visibility < 100% \( \alpha \neq 0, 180\textdegree \)
no entanglement (iris open)

entangled (iris 1mm)

- phase shift (866nm) < phase shift (870nm) \[ \therefore f = 1.7 \rightarrow 1 \]
- but phase shift < 45° to improve: walk-off compensation
- visibility < 100% (866nm, 870nm) \[ \therefore \alpha \neq 0, 180° \]
  to improve: group velocity compensation

frequency resolved photon pairs are entangled in polarization
(light source for WDM-QKD)

future: compensations of walk-off and group velocity (improve pol-entanglement)

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Beam-like photon pair generation for 2photon interference & polarization entanglement
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SPDC photon image

H: Horizontal
V: vertical

\[ |\Psi\rangle = \frac{1}{\sqrt{2}} [ |H_1\rangle |V_2\rangle + |V_1\rangle |H_2\rangle ] \]

Polarization Entangled photon pair

Beam-like photon pair

Crystal optic axis
Main idea (2photon interference)

\[ R_0 = 1 + r'(\Delta L')^2(\Delta L') \cos \left[ R_{\phi_0} \Delta L + \frac{r' - r}{2} \Delta L' + \Delta \phi \right] \]
Main idea (polarization entanglement)

\[ |\Psi\rangle = \frac{1}{\sqrt{2}} e^{i\phi} \left( |H_1\rangle |V_2\rangle + (e^{i\phi} |V_1\rangle) |H_2\rangle \right) \]

adjust phase

\[ |\Psi\rangle = \frac{1}{\sqrt{2}} \left( |H_1\rangle |V_2\rangle + |V_1\rangle |H_2\rangle \right) \]
HOM interference measurement (adjust path length)
2 photon interference by photon pair beams

classical lithography resolution ~ \( \lambda \)

2-photon interference (quantum lithography) 2 times higher resolution
polarization entanglement by photon pair beams

\[ |H_1\rangle|V_1\rangle + e^{i\phi}|H_2\rangle|V_2\rangle \]

rotate polarization 90 degrees by QWP plates

\[ |V_1\rangle|H_1\rangle + e^{i\phi}|H_2\rangle|V_2\rangle \]

(max entangle at \( \phi = n\pi \) n:integer)

measure coincidence scanning \( \phi \)

visibility = 0.90 ± 0.05 (highly entangled)
Our new scheme to generate photon pair beams for two purposes

- two-photon interference
- polarization entanglement

Our new scheme

resolution of 2-photon interference \((\lambda/2)\)

2 times higher than classical limit \((\lambda)\)

all photon pairs can be polarization entangled

efficient generation of polarization entangled pairs

cf.) traditional method: only crossing points of light cones

Beamlike photon-pair generation for two-photon interference and polarization entanglement

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Beamlike photon pairs were generated by spontaneous parametric down-conversion using a type-II β-BaB₂O₄ crystal. A pump laser generated photon pairs when it transmitted through the crystal and was reflected back into the crystal by a mirror to generate more photon pairs. The photon pairs generated when the pump laser first transmitted through the crystal (first photon pairs) were also reflected back into the crystal to overlap with the light path of the photon pairs generated in the second transmission of the pump laser through the crystal (second photon pairs). We observed interference between the first and second photon pairs modulated with a half period of the wavelength of the photon pairs, which demonstrates two-photon interference using the beamlike photon pairs. The fringe period confirms that the observed interference is not classical interference but quantum two-photon interference. Through rotating the angles of quarter-wave plates in the light paths of the photon pairs, we generated beamlike photon pairs with entangled polarization. The phase between the first and second photon pairs could be tuned by changing the position of mirrors reflecting the pump pulses and photon pairs. The fringes of coincidence counts showed that the beamlike photon pairs have polarization entanglement.

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You can find more detail information in this paper.
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on-going in National Chiao-Tung University
Thank you for your attention!