Synthetic gauge fields for ultracold neutral atoms

Yu-Ju Lin\textsuperscript{1,2}

Robert Compton\textsuperscript{2}, Karina Jimenez-Garcia\textsuperscript{2}, Trey Porto\textsuperscript{2}, William Phillips\textsuperscript{2} and Ian Spielman\textsuperscript{2}

\textsuperscript{1} Present address: Institute of Atomic and Molecular Sciences, Academia Sinica, Taiwan

\textsuperscript{2} Joint Quantum Institute, National Institute of Standards and Technology, Gaithersburg and University of Maryland

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Outline

• Introduction of ultracold quantum gases

  Review
  ✷ Bose-Einstein condensate (BEC)
  ✷ Light-atom coupling:
    trapping, manipulation, probe imaging

Main topics:
• To “charge” neutral atoms: synthetic gauge potentials $A^*$

• Uniform $A^*$ → $\vec{B}^* = \nabla \times \vec{A}^*$

• Spin-dependent $A^*$:
  can make non-abelian gauge potentials and spin-orbit coupling

  $[\vec{A}_i^*, \vec{A}_j^*] \neq 0$
Quantum gases

High Temperature $T$:
- thermal velocity $v$
- density $d^{-3}$
- "Billiard balls"

Low Temperature $T$:
- De Broglie wavelength $\lambda_{dB} = h/mv \propto T^{-1/2}$
- "Wave packets"

Even lower $T$: $\frac{N}{V} \lambda_{dB}^3 \geq 1$

quantum gases
Ultracold quantum gases

ultracold atoms: degenerate, non-classical gases \( \frac{N}{V} \lambda_{dB}^3 \geq 1 \)

(1) cold and dilute:

cold: s-wave scattering dominated: \( \lambda_{dB} > R_{vdw} \)
dilute: inter-particle spacing \( d > R_{vdw} \)
cold and dilute: reduced to contact interaction

\[
V(r-r') = \frac{4\pi\hbar^2 a}{m} \delta(r-r'), \quad a = s\text{-wave scattering length}
\]

Ex. Bose-Einstein condensate (BEC), Degenerate Fermi gas (DFG)

General references:
2. Making, probing, and understanding Bose-Einstein condensates, arxiv cond-mat 9904034
Review: Bose-Einstein condensate (BEC)

macroscopic occupation of a single-particle state $\varphi_0(x)$ exist for weakly interacting bosons: described by the order parameter $\psi(x) = \sqrt{N}\varphi_0(x)$

dynamics: Time dependent Gross-Pitaevskii Equation (TDGPE)

$$i\hbar \frac{\partial \psi(r,t)}{\partial t} = \frac{\hbar^2}{2m} \nabla^2 \psi(r,t) + V(r)\psi(r,t) + g|\psi(r,t)|^2 \psi(r,t)$$

interaction energy $g = \frac{4\pi\hbar^2a}{m}$
Dynamics of BEC

Second quantization
\[ \hat{\psi}(x) = \varphi_0(x)\hat{a}_0 + \sum_{i>0} \varphi_i(x)\hat{a}_i \]

Bogoliubov approximation
\[ \hat{a}_0 \rightarrow \sqrt{N} \]
\[ \hat{\psi}(x) = \sqrt{N} \varphi_0(x) + \delta\hat{\psi}(x) \]

\[ \hat{\psi}(r) \rightarrow \hat{\psi}(r,t) \]

Heisenburg equation of motion:
\[ i\hbar \frac{\partial \hat{\psi}(r,t)}{\partial t} = \left[ \hat{\psi}(r,t), \hat{H} \right] \]
\[ \hat{H} = \int dx \hat{\psi}^+(x) \left[ -\frac{\hbar^2}{2m} \nabla^2 + V(r) \right] \hat{\psi}(x) + \frac{1}{2} \int dr dr' \hat{\psi}^+(r) \hat{\psi}^+(r') V(r-r') \hat{\psi}(r') \hat{\psi}(r) \]

\[ \hat{\psi}(r,t) = \psi(r,t) + \delta \hat{\psi}(r,t) \]

\[ i\hbar \frac{\partial \psi(r,t)}{\partial t} = \frac{\hbar^2}{2m} \nabla^2 \psi(r,t) + V(r)\psi(r,t) + g|\psi(r,t)|^2 \psi(r,t) \]

Time dependent Gross-Pitaevskii Equation (TDGPE)

interaction energy
BEC in stationary ground state

Ground state: satisfies Gross-Pitaevskii Equation (GPE)

\[
-\frac{\hbar^2}{2m} \nabla^2 \psi(r) + V(r)\psi(r) + g|\psi(r)|^2 \psi(r) = \mu \psi(r)
\]

- GPE in the Thomas-Fermi (TF) limit: interaction >> kinetic energy

\[
|\psi(r)|^2 = \frac{1}{g} [\mu - V(r)]
\]

\[
g = \frac{4\pi\hbar^2 a}{m}, \ \mu = \text{chemical potential}
\]

TF BEC

\[
V(r) = \frac{1}{2} m\omega^2 r^2
\]

\[
|\psi(r)|^2 \quad \text{vs} \quad r
\]

\[\text{Na}/a_{bo} = 100\]
Ultracold quantum gases

ultracold atoms: degenerate, non-classical gases

✓ (1) cold and dilute: contact interaction \( V(r-r') = \frac{4\pi \hbar^2 a}{m} \delta(r-r') \)

(2) tunable interaction: Feshbach resonance

\[ a = \text{s-wave scattering length} \]

* can achieve strong interaction limit \( a \gg d \), even for a dilute gas
* can study strongly-correlated states with a simple model of interaction

Ex. BEC-BCS (Bardeen-Cooper-Schrieffer) crossover for fermions
Ultracold quantum gases

ultracold atoms: degenerate, non-classical gases

1. cold and dilute: contact interaction
   \[ V(r-r') = \frac{4\pi\hbar^2}{m} a \delta(r-r') \]

2. tunable interaction: Feshbach resonance

3. nearly disorder free
   precisely controlled magnetic and optical potentials
Magnetic potentials: Zeeman shift

An atom in an external magnetic field

Energy \[ E = -\mu \cdot \vec{B} \]

Force \[ \vec{F} = -\mu \cdot \nabla \vec{B} \]
Optical potentials: AC stark shift

Trapped atoms in light fields

Dipole moment \( \vec{d} = \alpha \vec{E} \)

Energy \( U_{\text{dip}} = -\vec{d} \cdot \vec{E} \)
\( \propto \alpha(\omega) I(r) \)

Red detuning \( \Delta < 0 \)

Blue detuning \( \Delta > 0 \)

Atoms are trapped in intensity maximum

Atoms are repelled from intensity maximum

Optical lattice

Laser
Ultracold quantum gases

ultracold atoms: degenerate, non-classical gases

1. cold and dilute: contact interaction

\[ V(r - r') = \frac{4\pi\hbar^2 a}{m} \delta(r - r') \]

2. tunable interaction: Feshbach resonance

3. nearly disorder free

precisely controlled magnetic and optical potentials

→ ideal for quantum simulation: model systems for condensed-matter physics
Ultracold atoms have realized iconic condensed matter systems

Use cold, degenerate gases: BEC (Bose-Einstein condensate) or degenerate Fermi gas

- **superfluid → Mott-insulator transition**: BEC in optical lattices

- **low dimensional systems**: 1D, 2D physics

- **BEC-BCS (Bardeen-Cooper-Schrieffer) crossover**: two-component Fermi gas, interaction tuned from repulsive → attractive

Ref: JILA, MIT, 2004
Review: Light-atom coupling

- For trapping, manipulating and probing atoms
- Dipole traps & optical lattices
- rf and Raman: couple between spin states
- Probe imaging
Light-atom interaction: two-level system

Rotating wave approximation (RWA): \( |\omega - \omega_0| \ll \omega_0 \)

\[
\Omega \cos \omega t = \frac{\Omega}{2} \left( e^{-i\omega t} + e^{i\omega t} \right)
\]

\[
|\psi(t)\rangle = c_1(t)e^{-i\omega_1 t}|1\rangle + c_2(t)e^{-i\omega_2 t}|2\rangle
\]

\[
c_1(t) \approx 1 \quad \text{for } c_1(0) = 1, \; c_2(0) = 0
\]

\[
c_2(t) \approx \frac{\Omega}{2} \left\{ \frac{1-e^{i(\omega_0-\omega)t}}{\omega_0-\omega} + \frac{1-e^{i(\omega_0+\omega)t}}{\omega_0+\omega} \right\}
\]

Expressed in rotating frame at \( \omega \):

\[
i\hbar \begin{pmatrix} \tilde{c}_1 \\ \tilde{c}_2 \end{pmatrix} = H_{rot} \begin{pmatrix} \tilde{c}_1 \\ \tilde{c}_2 \end{pmatrix} = \begin{pmatrix} -\Delta/2 & \Omega/2 \\ \Omega/2 & \Delta/2 \end{pmatrix} \begin{pmatrix} \tilde{c}_1 \\ \tilde{c}_2 \end{pmatrix}
\]

\( H_{rot} \) independent of \( t \)!
Light-atom interaction: two-level system

Rabi flopping: $c_1(t)=1$, $c_2(t)=0$

With damping:

$$|1\rangle \rightarrow |g\rangle, |2\rangle \rightarrow |e\rangle \quad \dot{\rho}_{22} = -\Gamma \rho_{22}$$

For $\Delta >> \Gamma, \Omega$:

$$U_{dip} = -d \cdot E = \hbar \frac{\Omega^2}{4\Delta}$$  \hspace{1cm} (dipole trap)

$$\Gamma_{sc} = \frac{\Omega^2}{4\Delta^2}$$  \hspace{1cm} (probe imaging)
Optical potentials: AC stark shift

Trapped atoms in light fields

Dipole moment

\[ \vec{d} = \alpha \vec{E} \]

Energy

\[ U_{\text{dip}} = -\vec{d} \cdot \vec{E} \]

Red detuning \( \Delta < 0 \)
Atoms are trapped in intensity maximum

Blue detuning \( \Delta > 0 \)
Atoms are repelled from intensity maximum

Optical lattice

\[ U_{\text{dip}} = \hbar \frac{\Omega^2}{4\Delta} \]
Probing atoms: absorption imaging

Absorption:

Optical density \( \text{OD} = \frac{I' - I}{I} \) for \( \text{OD} \ll 1 \)

\[
\text{OD} = \frac{\text{total scattered photon}}{\text{incoming probe photon}} = \frac{N\Gamma_{sc}}{IA/\hbar\omega}
\]

\[
= \frac{N\sigma}{A}
\]

on resonance:

\[
\sigma = \sigma_0 = \frac{3}{2\pi} \lambda^2
\]

off resonance:

\[
\sigma = \frac{\sigma_0}{1 + 4(\Delta/\Gamma)^2 + (I/I_{sat})}
\]

Light-atom coupling parameter:

- intensity: \( I/I_{sat} \)
- detuning: \( \Delta/\Gamma \)

\[
I_{sat} = \Gamma \hbar\omega/2\sigma_0
\]

\( \Gamma_{sc} \): scattered photon# per atom

\( \sigma \): scattering cross section per atom

\( \Omega = d^* E \)
Time-of-Flight (TOF) imaging

• Switch off trap, free expansion

• measure $k$ distribution : $k$ mapped to $x$
  (1) ballistic expansion: no interaction during TOF
  (2) after long expansion $t \gg 1/\omega$

• thermal atoms: ballistic expansion
BEC: superfluid expansion, interaction driven
Detection of BEC: bimodal distribution at $T>0$

$\text{thermal} \rightarrow \text{thermal} + \text{BEC} \rightarrow \sim \text{pure BEC}$

Distribution in $(k_x, k_y)$ space

Long expansion time $t \gg 1/\omega$
Detection of BEC: anisotropic distribution
Ultracold atoms have realized iconic condensed matter systems

Use quantum gases: BEC (Bose-Einstein condensate) or degenerate Fermi gas

- superfluid $\rightarrow$ Mott-insulator transition: BEC in optical lattices
  
  ![Superfluid to Mott-Insulator Transition](image)

  Ref: Greiner et al., 2003

- low dimensional systems:
  - 1D: Tonks-Giradeau gas
    
    ![1D Tonks-Giradeau Gas](image)

    Ref: D. S. Weiss, 2004
  - 2D: BKT superfluid
    
    ![2D BKT Superfluid](image)

    (Berezinskii-Kosterlitz-Thouless superfluid)

    Ref: Jean Dalibard, 2006
Ultracold atoms have realized iconic condensed matter systems

Use cold, degenerate gases: BEC (Bose-Einstein condensate) or degenerate Fermi gas

- **superfluid → Mott-insulator transition**: BEC in optical lattices

- **low dimensional systems**: 1D, 2D physics

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Ref: JILA, MIT, 2004
What we want to simulate with ultracold atoms
to “charge” neutral atoms by creating a “ synthetic gauge potential $A^*$”

- new approach to generate large $B^*$ to study quantum-Hall physics:
  (2D system and $\nu = \frac{N_{2D}}{N_v} \leq 1$ \(N_{2D} = \) atom#, \(N_v = \) # of flux quanta)

Advantages to study w/ ultracold atoms
- bosonic $\nu = 1$ state: w/ binary contact interaction, nonabelian, for topological quantum computation
  Ref: N. R. Cooper, 2008

- Spin-dependent $A^*$: spin-orbit coupling
  TR preserved topological insulators, topological superconductors
Outline: synthetic gauge potentials $A^*$

Raman-dressed BEC

BEC

B\textsubscript{real}\\ \\
$B^* = \nabla \times A^*$

Spin dependent $A^*$: spin-orbit coupling

superfluid in $B^*$ (like superconductor in B)

Magnetic field $B^* = \nabla \times A^*$

$H = \frac{\hbar^2}{2m^*} \left(\frac{k_x - qA^*_x}{\hbar} \right)^2 + k_y^2$

Vector potential $qA^*/\hbar k_L$

Detuning $\Delta/E_L$

Spin states $|\uparrow\rangle$, $|\downarrow\rangle$

Quasimomentum $q/k_L$

Minima location in units of $k_L$
Introduction of gauge potential

• Optically induced vector gauge potential $A^*$ for neutral atoms:

\[ H = \frac{(p - q^* A^*)^2}{2m^*} + V(x) \]

→ synthetic electric and magnetic fields

\[ E^* = -\frac{\partial A^*}{\partial t}, \quad B^* = \nabla \times A^* \]

• Create synthetic field $B^*$ for neutral atoms: effective Lorentz force
to simulate charged-particles in real magnetic fields

• Light-induced potential to generate $B^*$ in lab frame, no rotation of trap:
(1) steady $B^*$, not metastable
(2) easy to add optical lattices

$^87\text{Rb}$
Traditional methods to create $B^*$: rotation

rotating neutral atoms

$$F_{\text{Coriolis}} = 2m\Omega v_{\text{rot}}$$

$$H_{\text{rot}} = \frac{\hbar^2}{2m} \left[ (k_x - \frac{m\Omega y}{\hbar})^2 + (k_y + \frac{m\Omega x}{\hbar})^2 \right] + V'(r)$$

$$V'(r) = \frac{1}{2} m(\omega^2 - \Omega^2)r^2$$

$N_v$ vortices, $L/N = N_v/2$ (large $N_v$)

$\omega/2\pi \sim 10\text{Hz}$, $\Omega/\omega = 0.975$

$N \sim 10^6$, $R \sim 30\mu\text{m}$

Coddington et al., JILA, 2004

charge $q$ in $B$

$$F_{\text{Lorentz}} = qvB$$

$$H_B = \frac{\hbar^2}{2m} \left[ (k_x - \frac{qBy}{2\hbar})^2 + (k_y + \frac{qBx}{2\hbar})^2 \right] + V(r)$$

$$V(r) = \frac{1}{2} m\omega^2 r^2$$

$N_v$ vortices or flux quanta

(one vortex $\leftrightarrow \Phi_0 = h/q$)

rotating BEC (experiment)

w/ mean field interaction
Principles (I)

- charged particle \( q \) in a real field \( \vec{B} = B\hat{z} \), Landau gauge

\[
H_B = \frac{\hbar^2}{2m} \left[ (k_x - \frac{qA_x}{\hbar})^2 + k_y^2 \right]
\]

\[
\delta k_x \equiv \frac{qA_x}{\hbar} = \frac{qBy}{\hbar}
\]

\( \vec{B} = \nabla \times \vec{A} \)

- to simulate w/ laser-atom interaction

- laser photons: create \( \delta k_x \) = momentum shift along \( x \)
  \( \rightarrow \) make \( \delta k_x(\Delta) \), \( \Delta \)=laser-atom detuning

  \( \rightarrow \) make \( \Delta = \Delta' y : \delta k_x(y) \)

  \( \rightarrow \) synthetic field \( \frac{q^* B^*}{\hbar} = \frac{\partial (\delta k_x)}{\partial y} \) along \( z \)
Principles (II):
Formalism of Light-atom Coupling

• controlled Raman detuning $\Delta = \Delta \omega_L - g \mu_B B$, $B=B_0-b'y$ or $B=B_0(t)$

$$\hat{H} = \frac{\hbar^2 \hat{k}^2}{2m} + V(x) + H_{\text{int}}(\Omega_R, \Delta) = \begin{pmatrix}
-1, k_x + 2 \\ 0, k_x \\ +1, k_x - 2
\end{pmatrix}
= \begin{pmatrix}
(\hat{k}_x + 2)^2 - \Delta & \Omega_R / 2 & 0 \\
\Omega_R / 2 & \hat{k}_x^2 - \varepsilon & \Omega_R / 2 \\
0 & \Omega_R / 2 & (\hat{k}_x - 2)^2 + \Delta
\end{pmatrix} + k_y^2 + V(x)

\text{k (k}_L\text{), E(E}_L\text{)}
\text{E}_L = \hbar^2 k_L^2 / 2m$$

• diagonalize $\rightarrow$ Raman-dressed state: eigenvalue $= E_0(\Delta) + (k_x - \delta k_x(\Delta))^2$
Principles (III): Shift of momentum in dispersion relation

- the lowest energy dressed-state:
  \[ \delta k_x = \frac{q^* A_x^*}{\hbar} = \text{momentum shift} \]
  \[ H_x = \frac{\hbar^2}{2m} \left[ (k_x - \frac{q^* A_x^*}{\hbar})^2 \right] \]
Setup: BEC production

- load MOT from Zeeman slower: $\sim 10^9$ atoms in 3 s
- rf-evaporative cooling in a quadrupole magnetic trap for 3 s, $|F=1, m_F=-1\rangle$
- single beam optical dipole trap + weak magnetic trap: evaporate in hybrid potential for $\sim 7$ s $\rightarrow 2 \times 10^6$ atoms in BEC
- load the BEC into the crossed dipole trap: $5 \times 10^5$ atoms
- total cycle time $\sim 15$ s
Actual experimental Setup
Setup: Raman-dressed BEC

- Bose-Einstein condensate in a crossed dipole trap, $^{87}\text{Rb } |F=1, m_F = -1\rangle$
  - $N \approx 5 \times 10^5$ every 15 s
- Trap frequency: $(\omega_x, \omega_y, \omega_z) \approx 2\pi \times (70, 70, 80)$ Hz
- For $N \rightarrow 2.5 \times 10^5$ in Raman-dressed state
  - from decreasing dipole trap power to reduce heating from Raman beams
- Zeeman shift: linear = $g\mu_B B = 2.71$ MHz, quadratic = 1.0 kHz
  - $\lambda = 801.7$ nm (D1=795nm, D2=780nm)
Adiabatic loading: uniform $A^*$ (I)

- **(a)** Bare state, $|\pm 1\rangle$
- **(b)** rf-dressed state, $\delta k_x = 0$
- **(c)** rf+Raman-dressed state, $\delta k_x = 0$
- **(d)** Raman-dressed state, $\delta k_x = 0$
- **(e)** Raman-dressed state, $\delta k_x(\Delta) = q^* A^*/\hbar \neq 0$

Time-of-Flight images of $|-1,k_x+2\rangle$, $|0,k_x\rangle$, $|+1,k_x-2\rangle$

measure $\delta k_x$ from spin $|0,k_x = \delta k_x\rangle$

- **(b)(c)** $\Delta = 0$, $\delta k_x = 0$
- **(d)** $\Delta \neq 0$, $\delta k_x \neq 0$

Quasi-momentum $k_x/k_L$

$\langle \hat{x} \rangle_{m_F} = 0 !$
Uniform vector potential $A^*$ vs. detuning $\Delta$

effective vector potential $q^*A^*/\hbar = $ measured quasi-momentum $k_x$
adiabatic loading at energy minimum $\rightarrow k_x=q^*A^*/\hbar$

vector potential $q^*A^*/\hbar$ vs. detuning $\Delta$
Uniform vector potential $A^\ast$ vs. detuning (II)

- for weak coupling $\Omega_R < 4.5 \, E_L$, double energy minimum in k space
Synthetic field $B^* = \nabla \times A^*$

**detuning (at center)**

**detuning gradient**

**Raman**

$0 (a) (b) (c) (d) (e')$

$\Delta' = d\Delta/dy$

$B = B_0 - b'y$

**TOF imaging**

Time-of-Flight images of $|{-1, k_x+2}\rangle$, $|0, k_x\rangle$, $|{+1, k_x-2}\rangle$

- $\Delta = \Delta\omega_L - g\mu B$
- $\Delta' = q' A' (\Delta)/\hbar$

**Spin $m_F (y)$**

(a) \hspace{2cm} \hspace{2cm} (b)(c) \hspace{2cm} (d) $B^* = 0$, $A^* = 0$ \hspace{2cm} (e') $B^* \neq 0$ along $z$, $A^* (y)$

**Quasi-momentum $k_x/k_L$**

$\delta k_x = q' A' (\Delta)/\hbar$
Raman-dressed BEC in synthetic B*

Spin and momentum projection, TOF=25.1ms

\[ H_B = \frac{\hbar^2}{2m^*} \left( k_x - \frac{q^* B^* y}{\hbar} \right)^2 + \frac{\hbar^2 k_y^2}{2m} + V(x, y) \]

- vector potential \( \delta k_x = \frac{q^* A^*}{\hbar} = \frac{q^* B^* y}{\hbar} \)

- magnetic field B*

\[ \frac{q^* B^*}{\hbar} = \frac{\partial (\delta k_x)}{\partial y} = \Delta \frac{\partial (\delta k_x)}{\partial \Delta} \]

detuning gradient \( \Delta = \frac{d\Delta}{dy} \)

A* vs. \( \Delta \) or y

B* vs. \( \Delta \) or y

2R_BEC = 18 \( \mu \text{m} \)
Vortex number $N_v$ vs. $B^*$

- For infinite system size:
  
  $$N_v = \text{area} \times \frac{B^*}{\hbar q^*} = \Phi_{B^*} \div (\hbar q^*)$$

- Threshold energy $E_v$ to create $N_v=1$

  for finite system radius $R$: $E_v \propto 1/R^2$

Conclusions and outlook

- Observing vortices in a Raman-dressed BEC: superfluid in a synthetic magnetic field $B^*$

- Stable $B^*$ in lab frame, easy to add optical lattices

- Outlook: optimize vortex density ($\propto B^*$) and trap geometry

- Hofstadter butterfly: add 2D lattices

- Long term: large $B^*$ in quantum Hall regime
  prepare 2D systems, with filling factor $\nu = N_{2D}/N_v \leq 1$

  1D lattice along z, compress along y, relax along x
  (small $N_{2D}$, large $B^*$ and $N_v$)